

# Quantum Chemistry & Quantum Computing

*Saad Yalouz*

*CNRS, Laboratoire de Chimie Quantique de Strasbourg  
Institut de Chimie de Strasbourg*



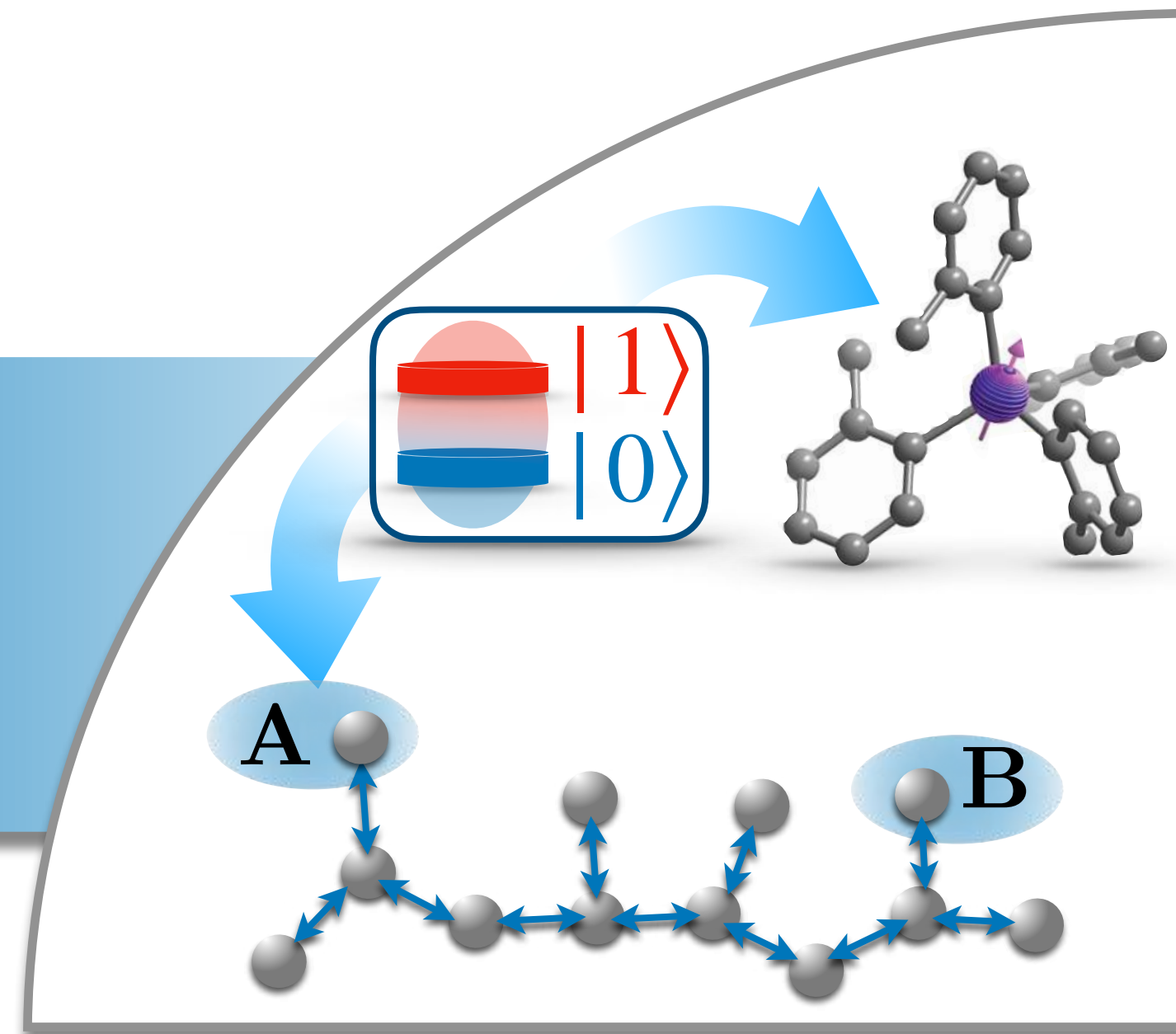
# Quantum Chemistry & Quantum Computing

**$\langle$  LCQ | S  $\rangle$**

Laboratoire de Chimie Quantique de Strasbourg

# Quantum Chemistry & Quantum Computing

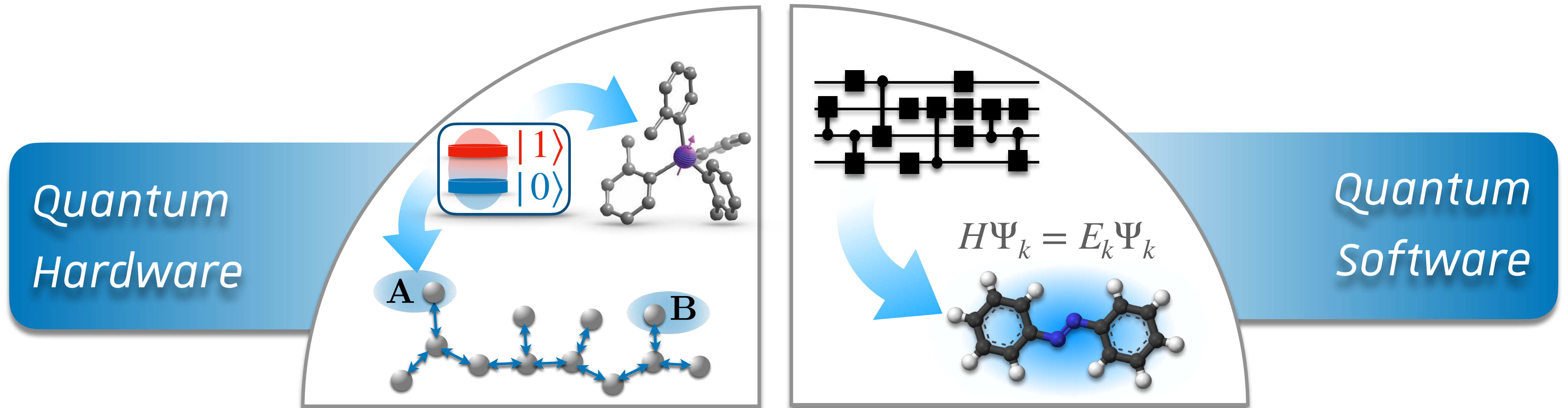
Quantum  
Hardware



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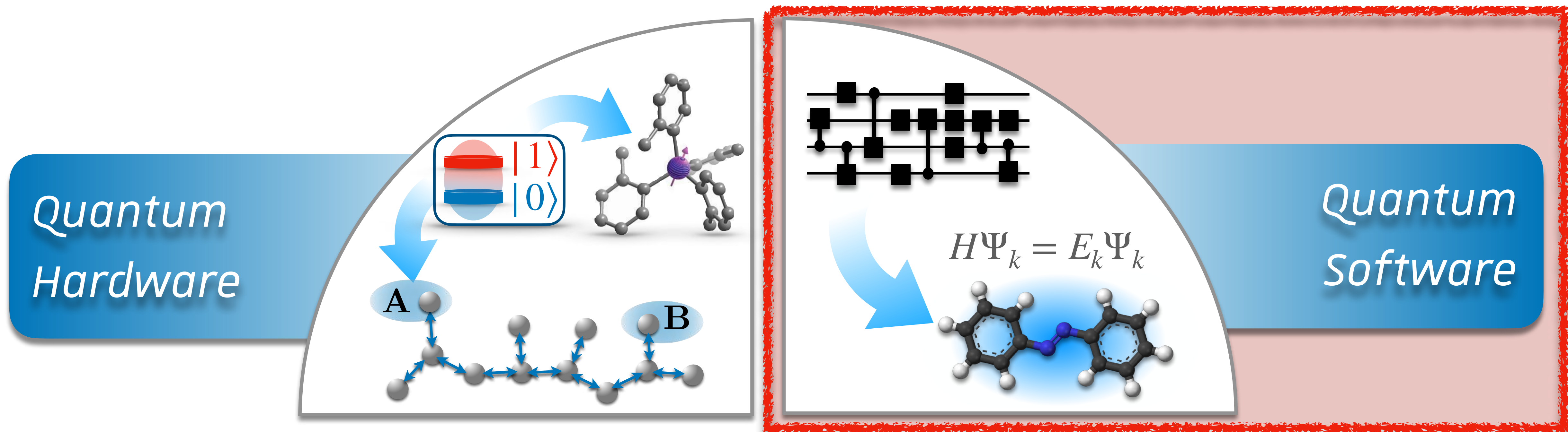
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# Quantum Chemistry & Quantum Computing



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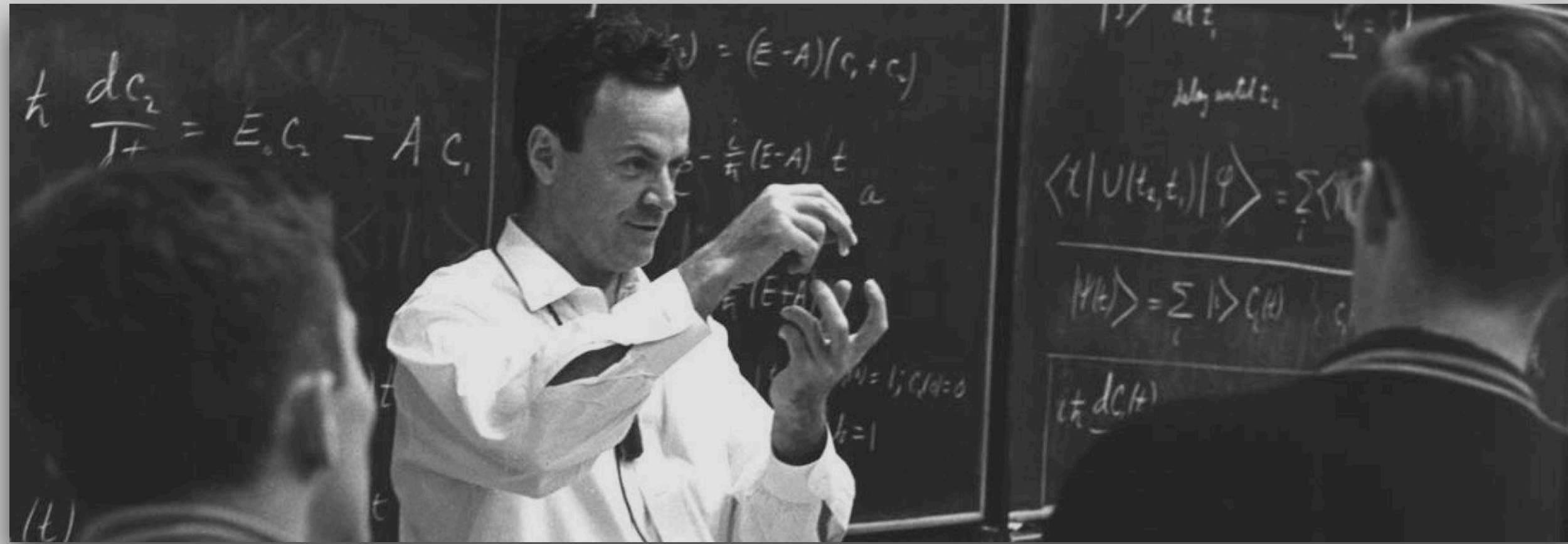
Laboratoire de Chimie Quantique de Strasbourg

# Summary

- I) Introduction to quantum computing
- II) From quantum computing to chemistry
- III) Quantum algorithm for photochemistry
- IV) Take home messages

# I) Introduction to quantum computing

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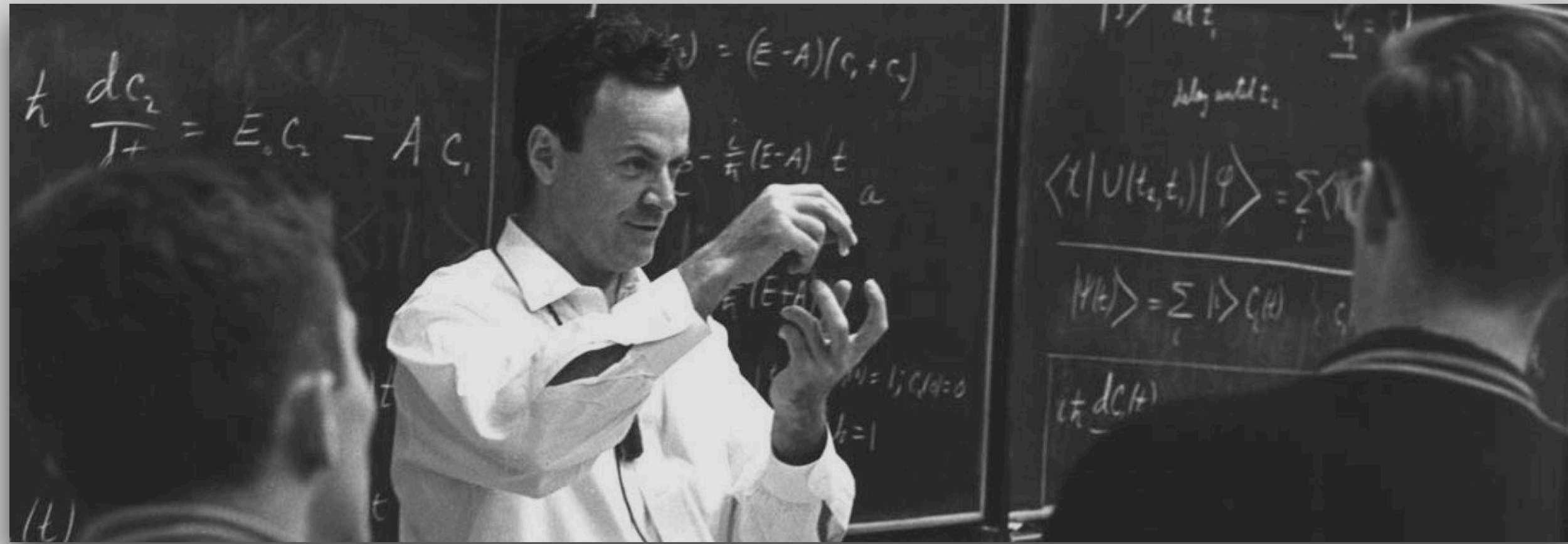


**-Richard P. Feynman**

*“Nature (e.g. atoms, molecules ...) isn't classical and if you want to make a simulation of nature, you'd better make it quantum mechanical.”*



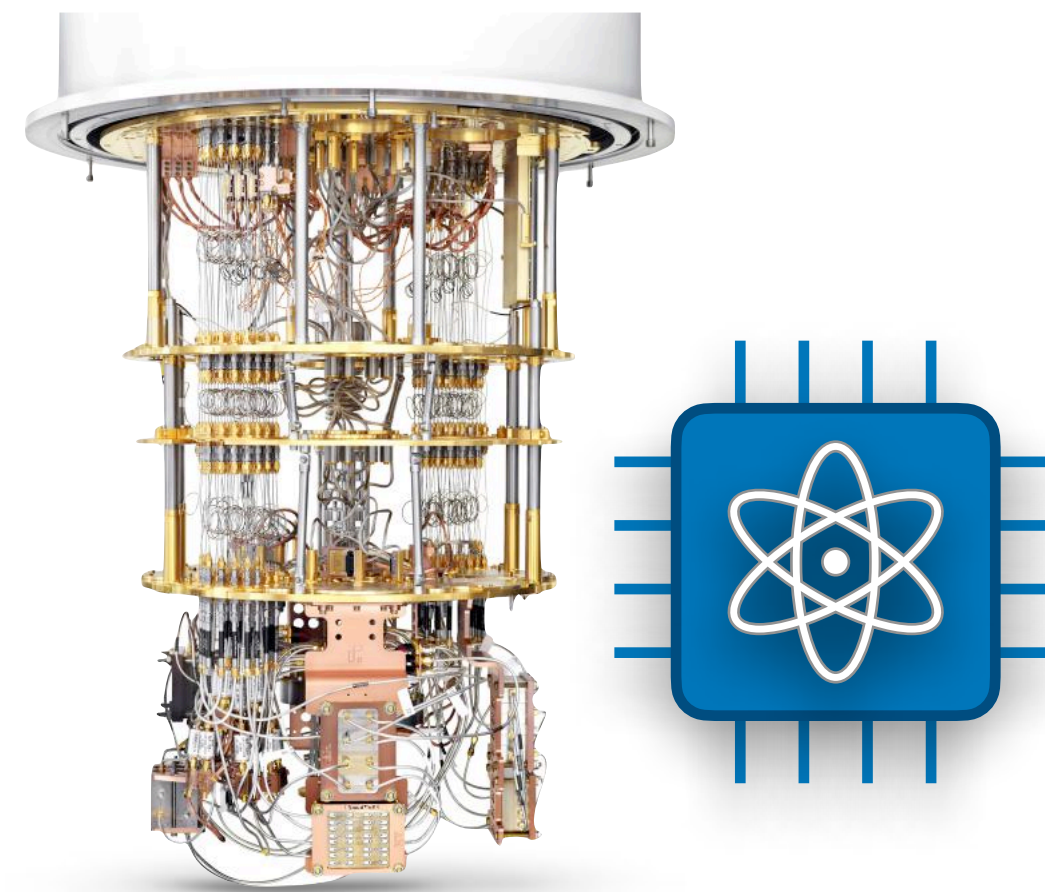
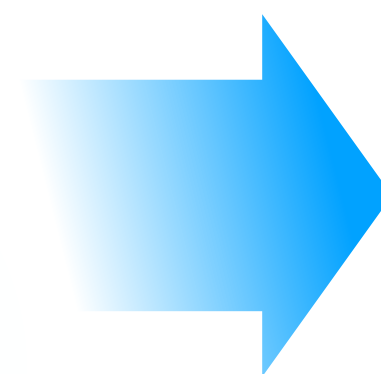
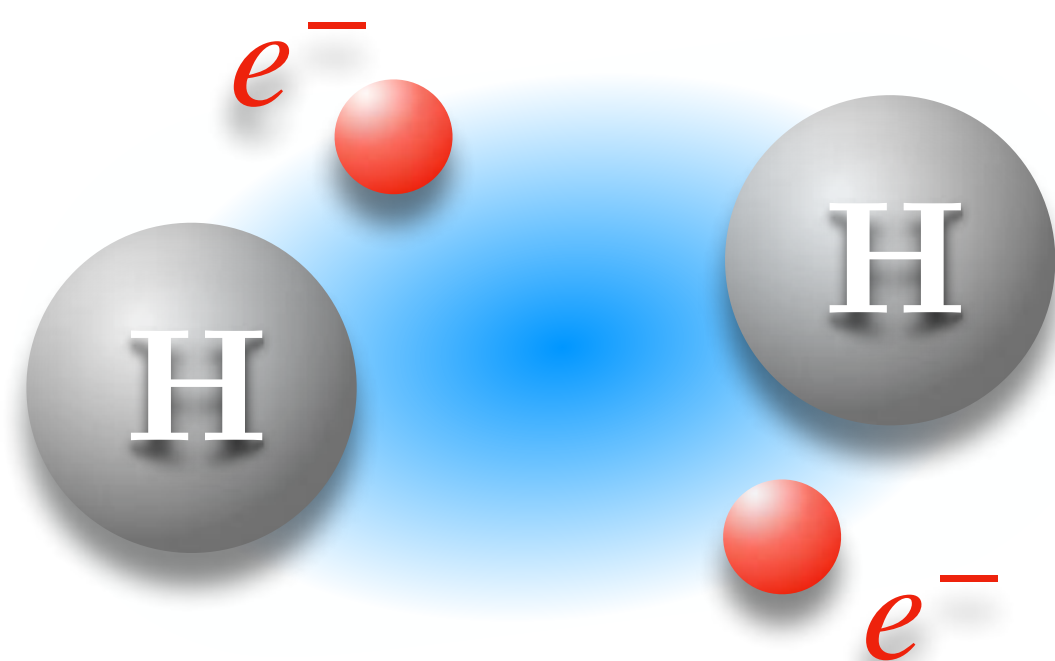
# I) Introduction to quantum computing



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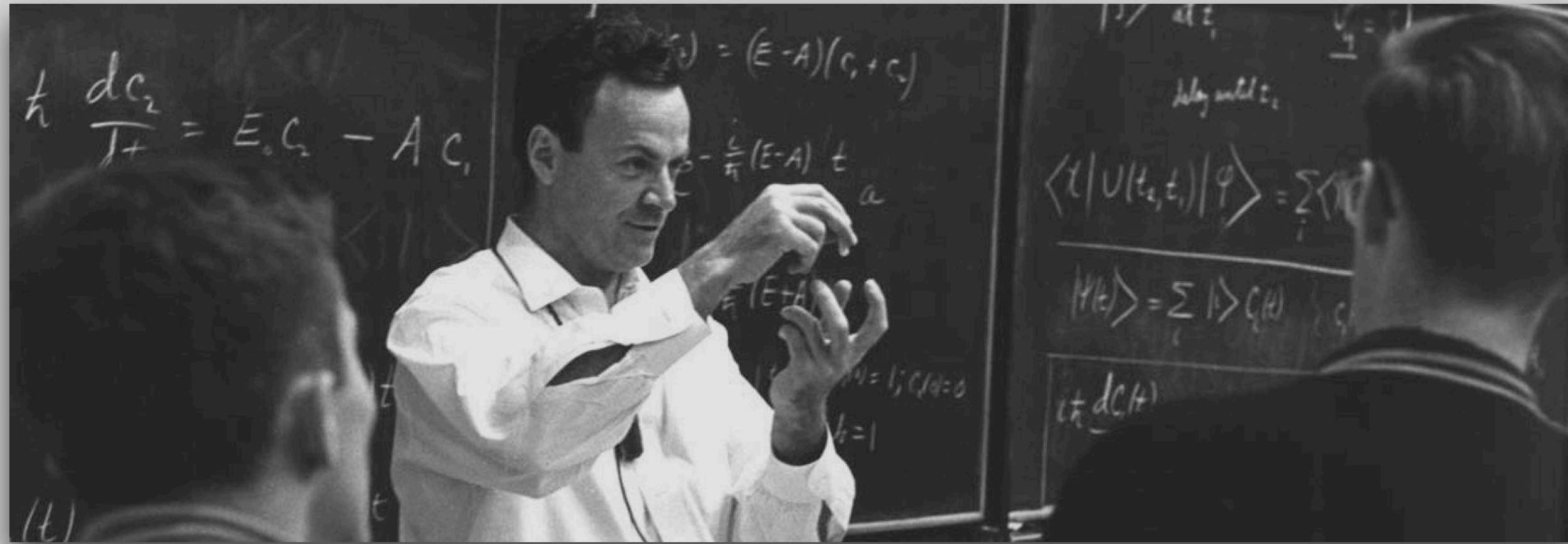
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Quantum Chemistry



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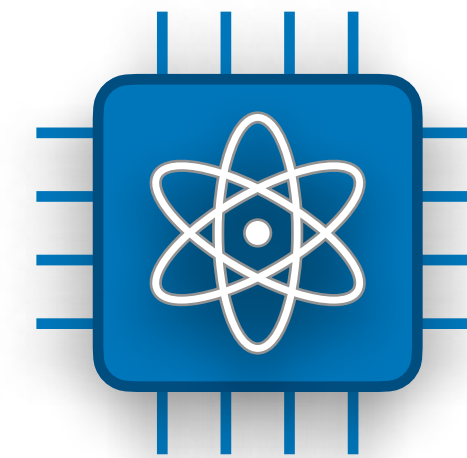
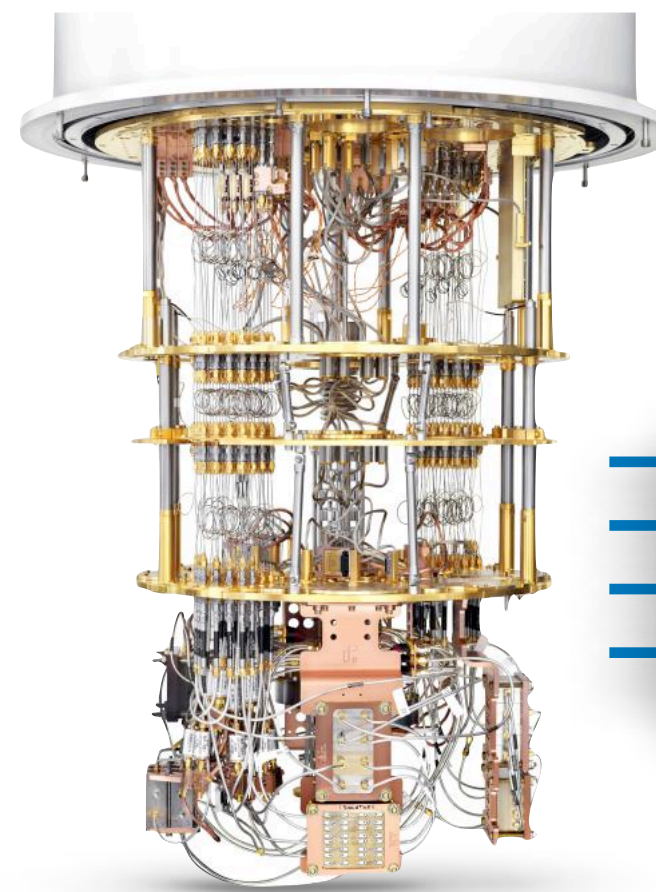
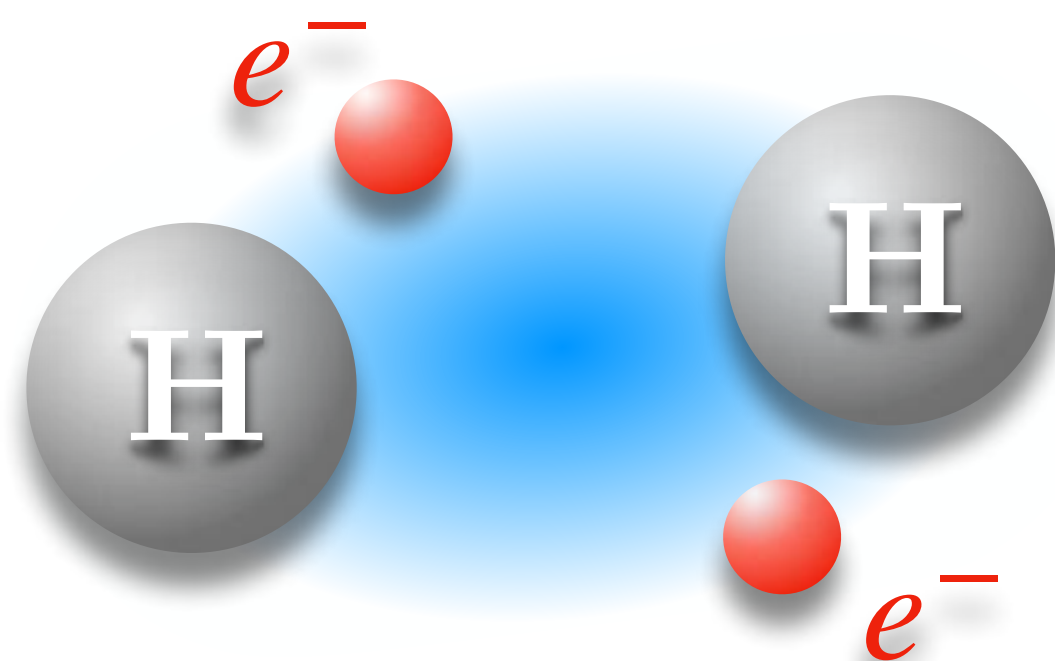
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*Quantum Computer*

VS.



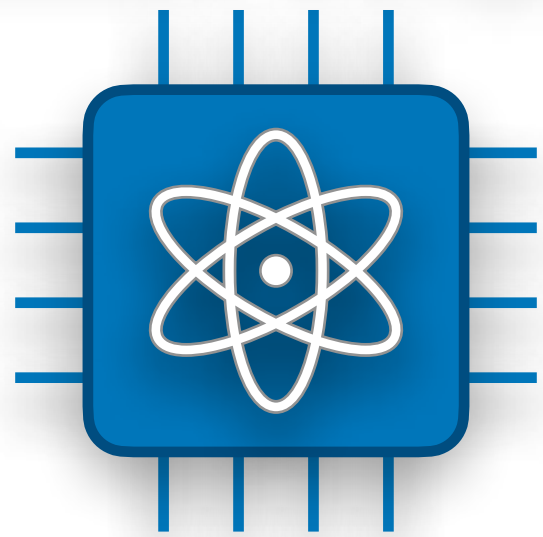
*Classical Computer*

# I) Introduction to quantum computing

*Classical Computer*



*Quantum Computer*



# I) Introduction to quantum computing

Unit of Information

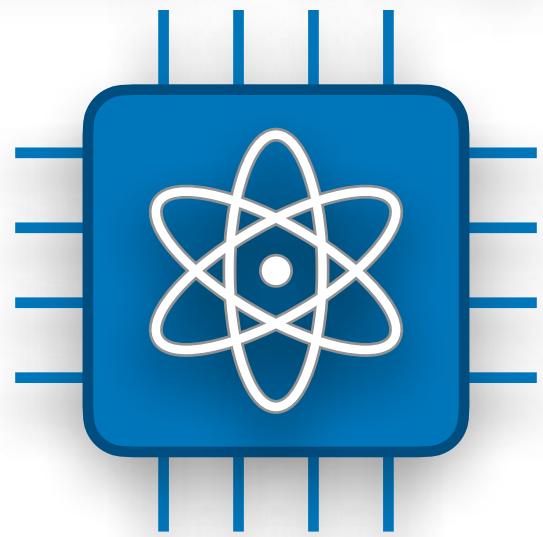
Basic Logic

Prog. Language

*Classical Computer*



*Quantum Computer*



# I) Introduction to quantum computing

Unit of Information

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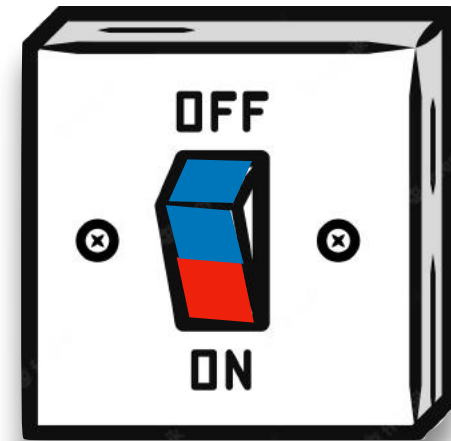
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The Bit

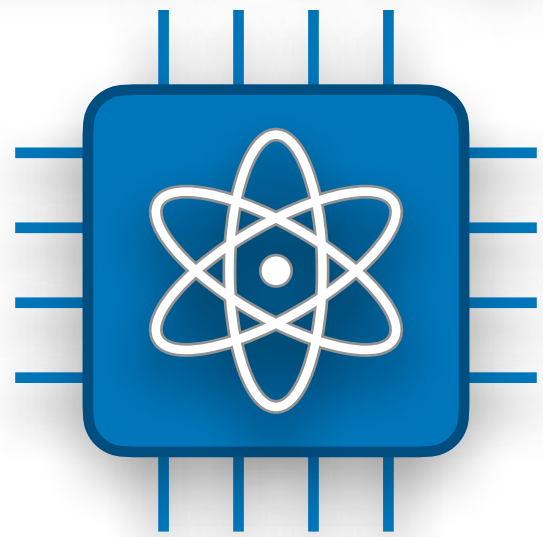
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1



VS.

*Quantum Computer*



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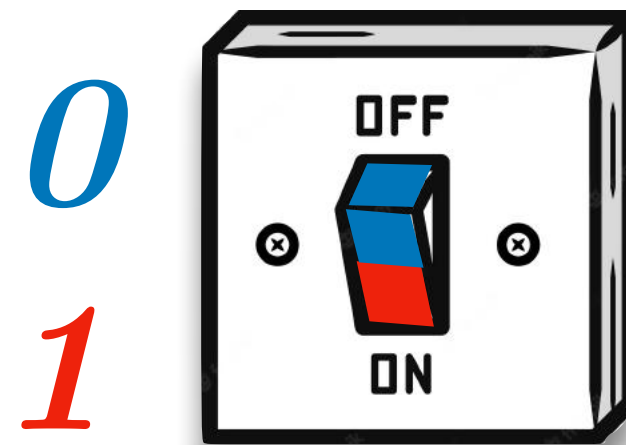
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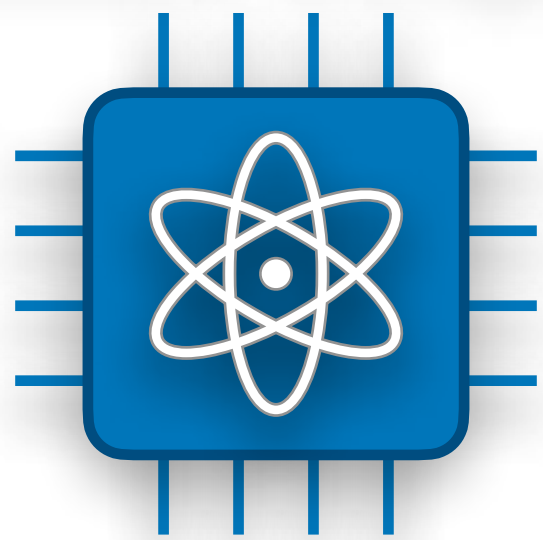


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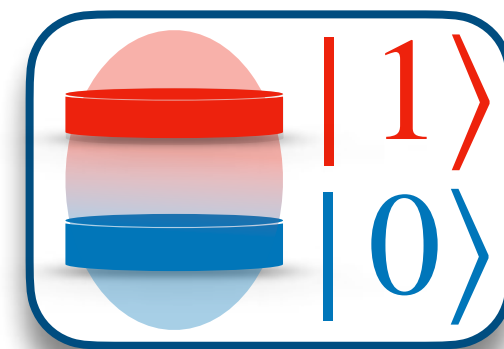


VS.

*Quantum Computer*



The Qubit



$$|Q\rangle = c_0 |0\rangle + c_1 |1\rangle$$

# I) Introduction to quantum computing

Unit of Information

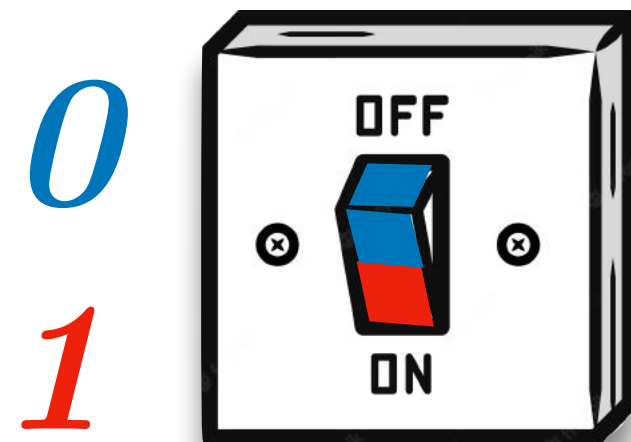
Basic Logic

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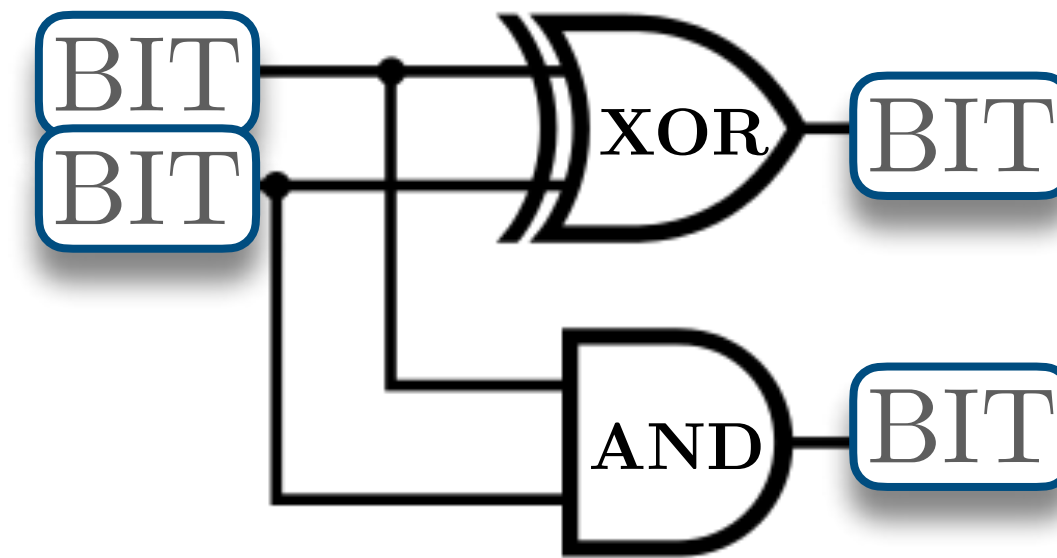
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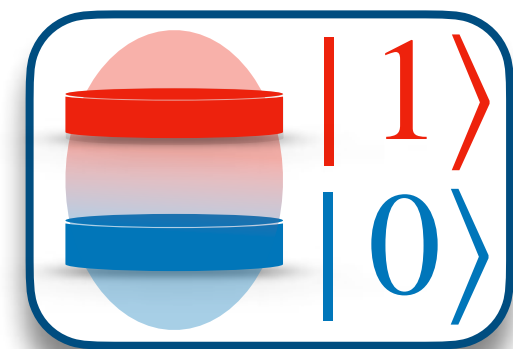


Logical circuit



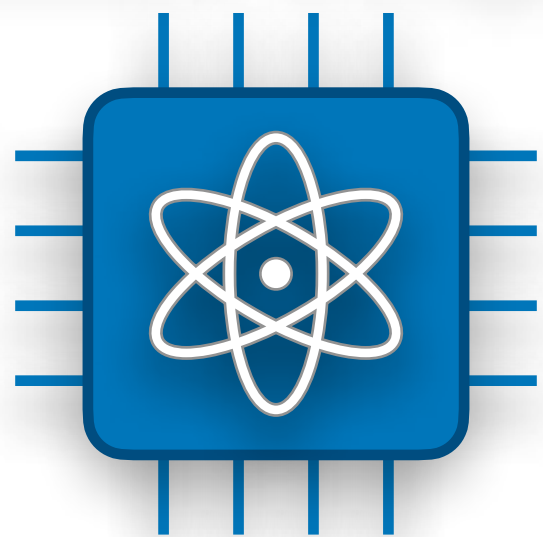
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*Quantum Computer*



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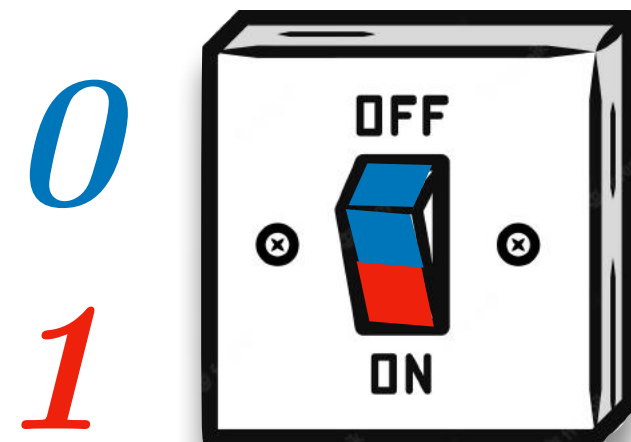
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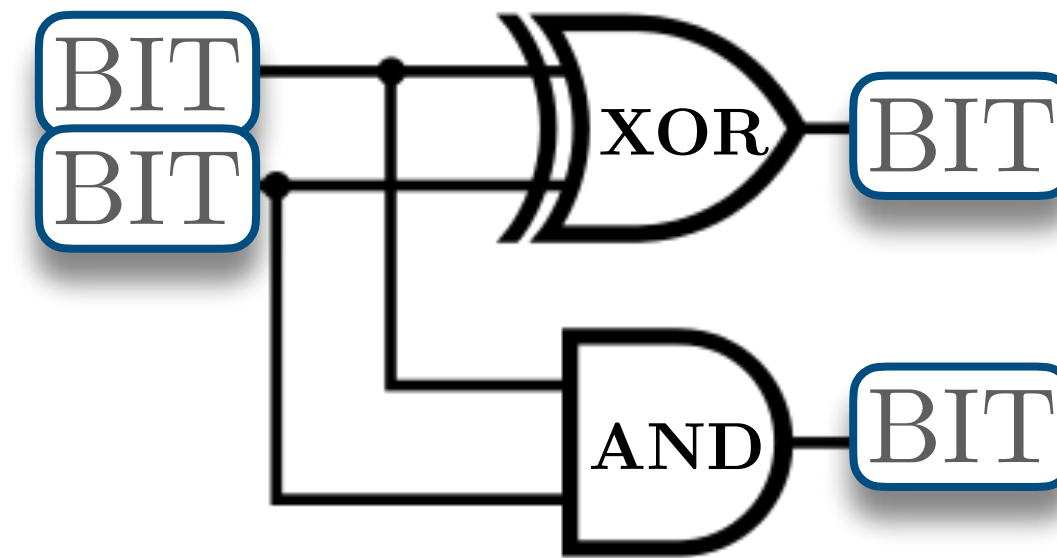
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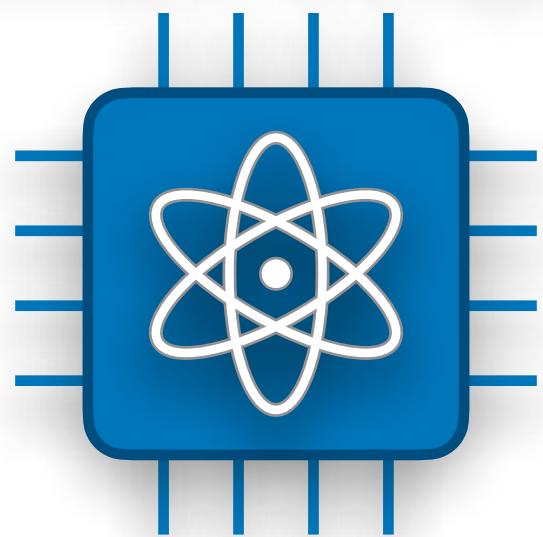


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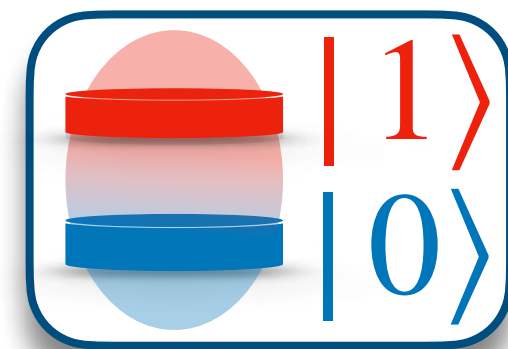


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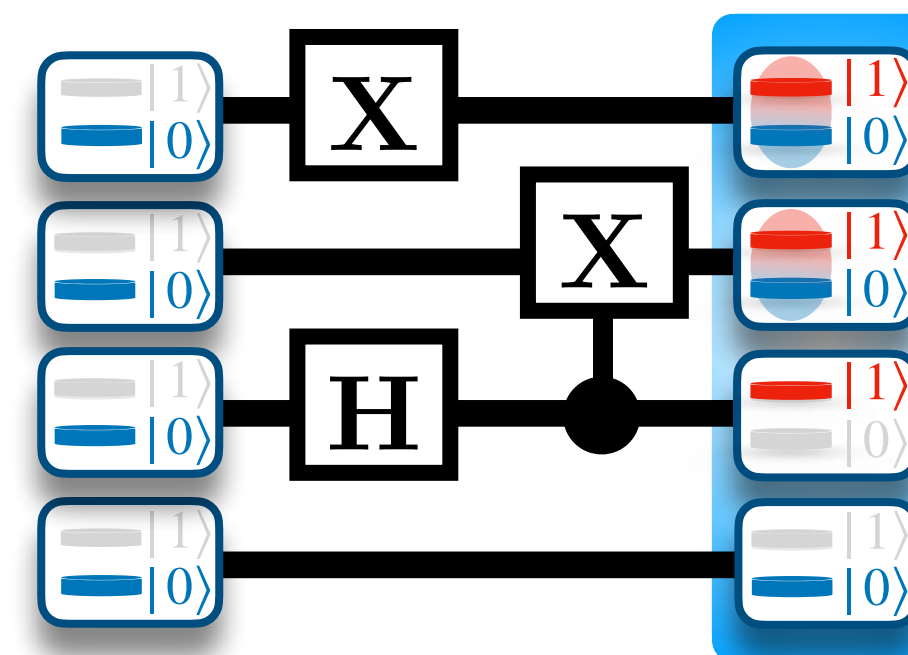


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Quantum Circuit





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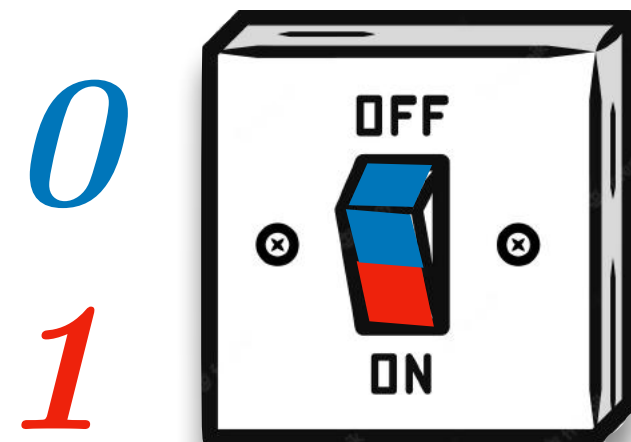
Basic Logic

Prog. Langage

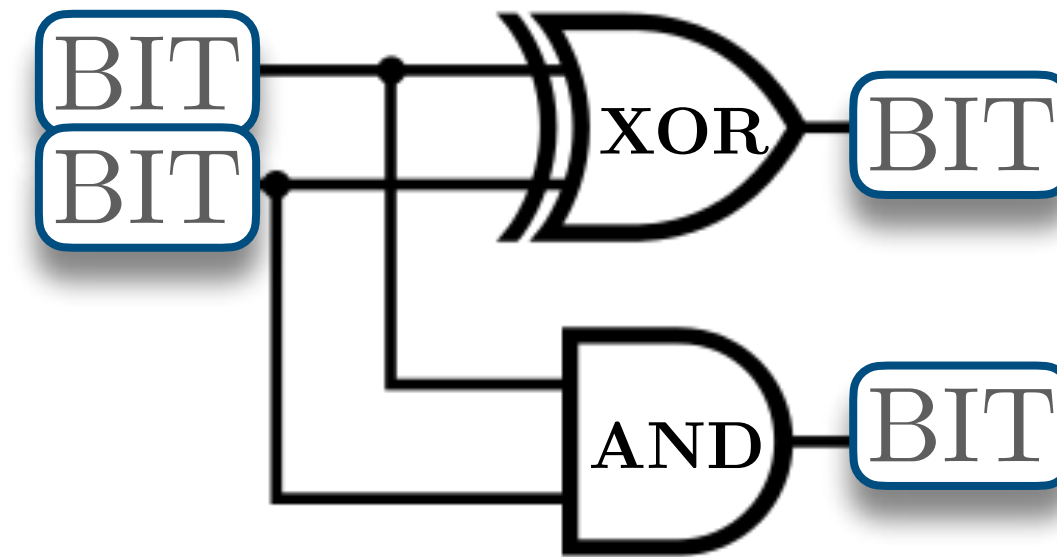
Classical Computer



The Bit



Logical circuit

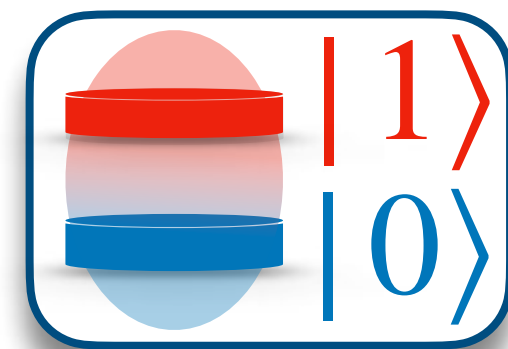


Fortran, C, Python ...



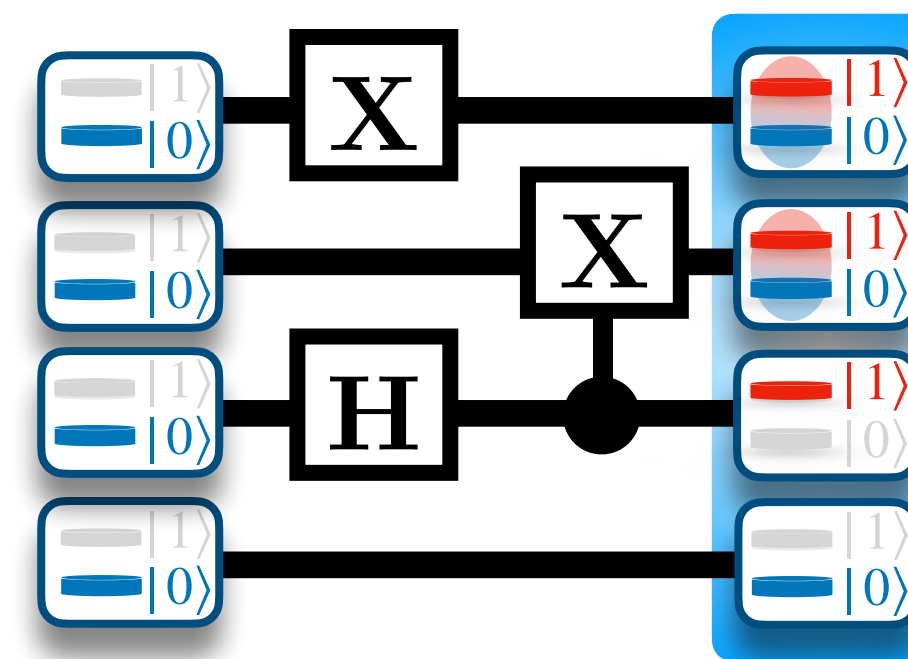
VS.

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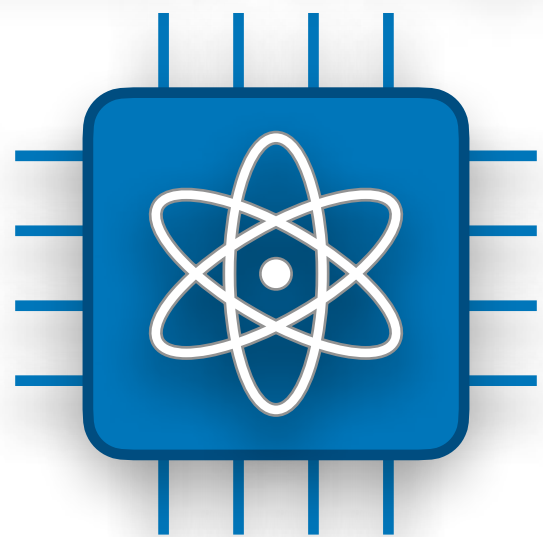


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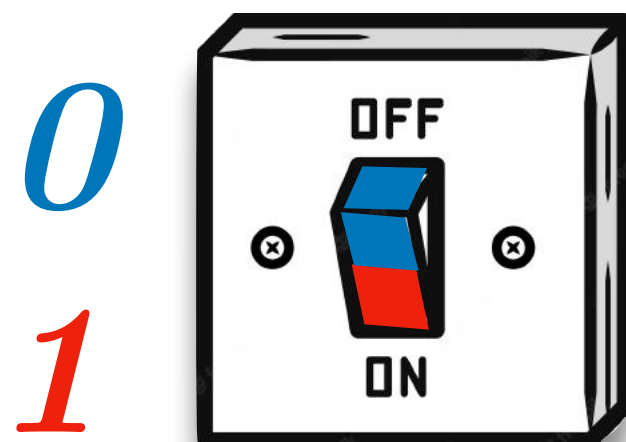
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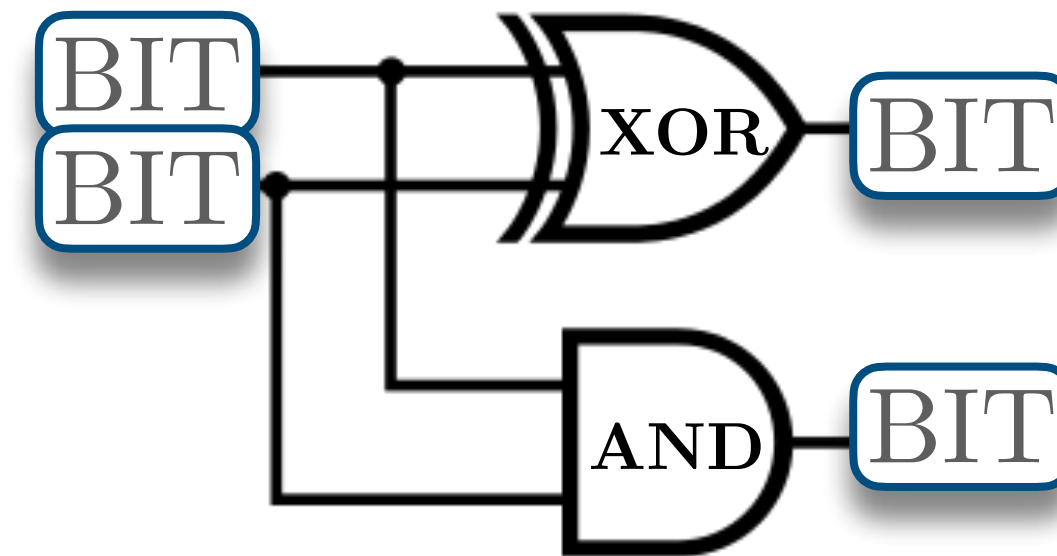
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Logical circuit

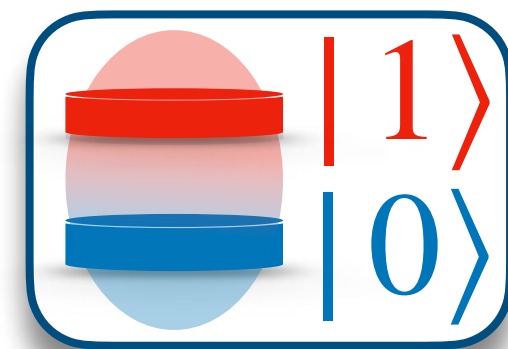


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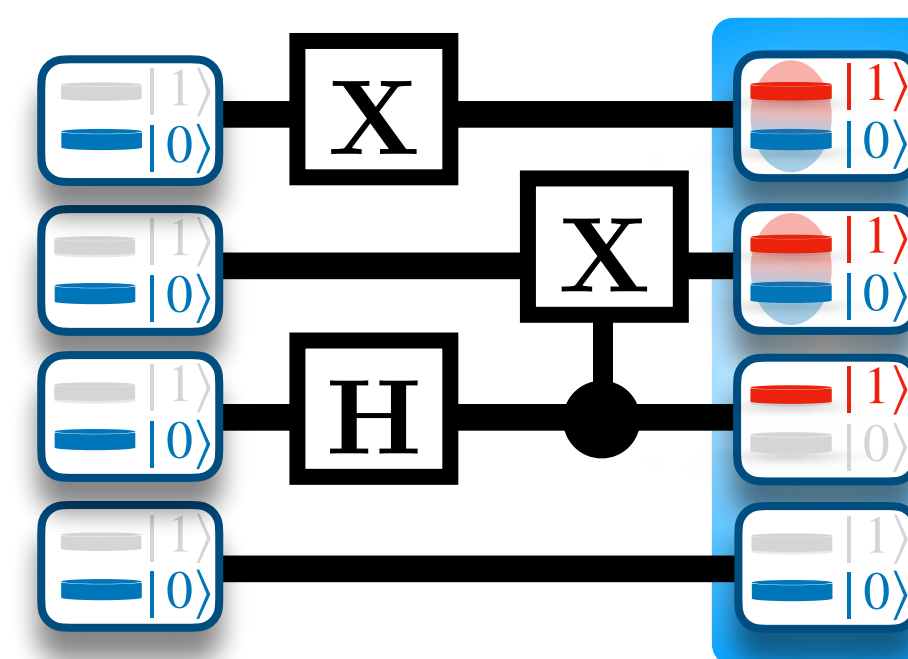
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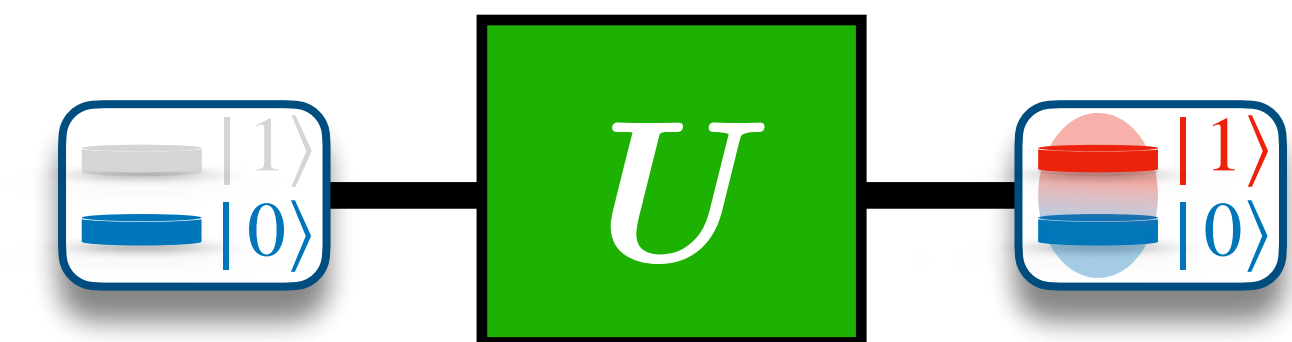


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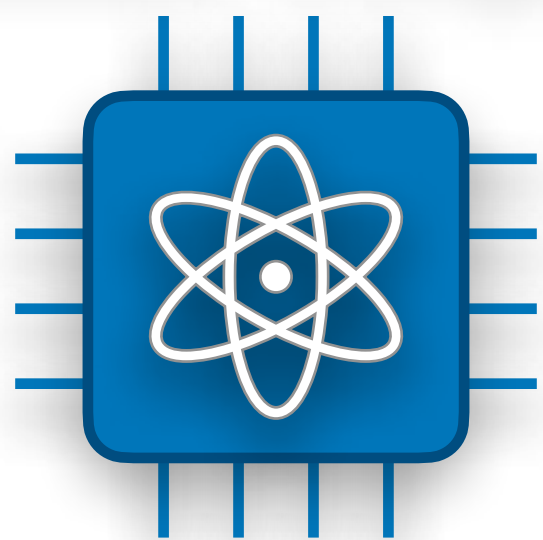
Quantum Circuit



Quantum Physics !  
(Unitary Transformations)



Quantum Computer



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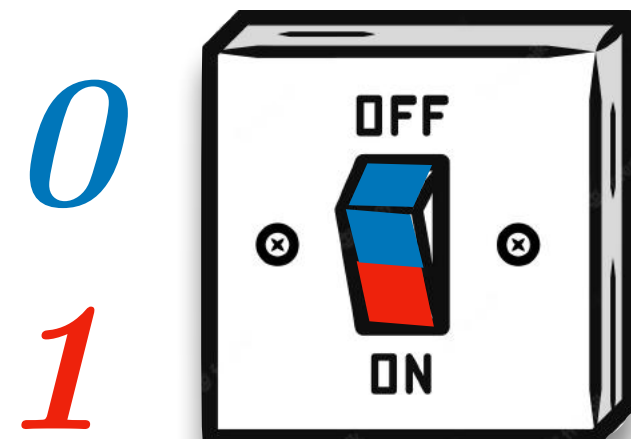
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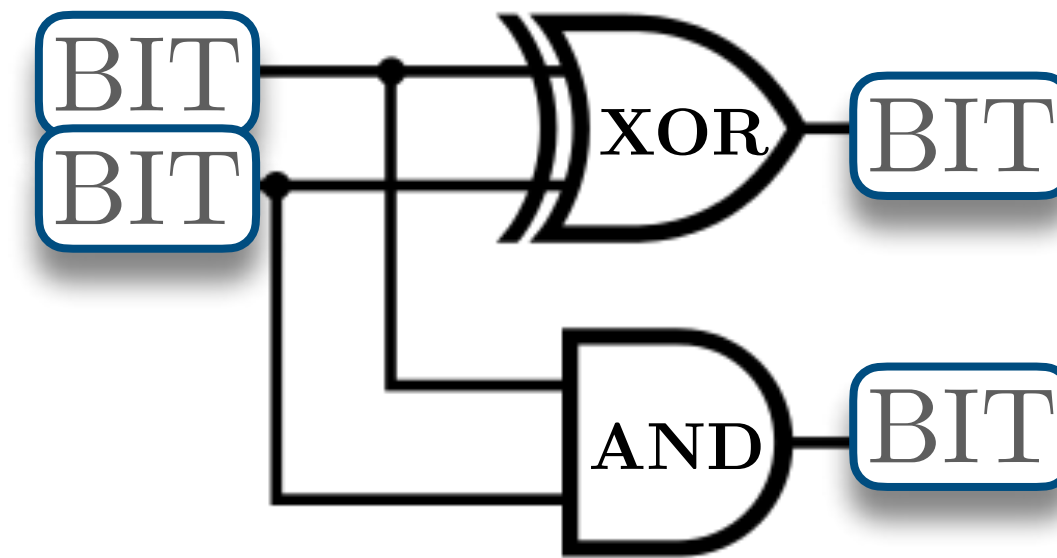
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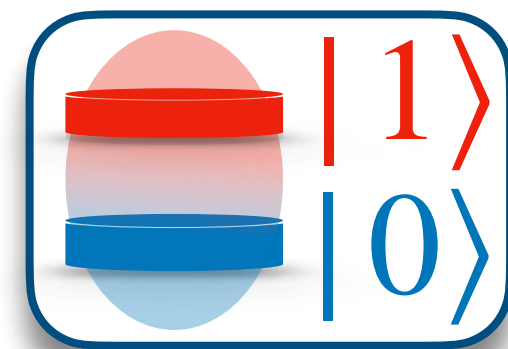


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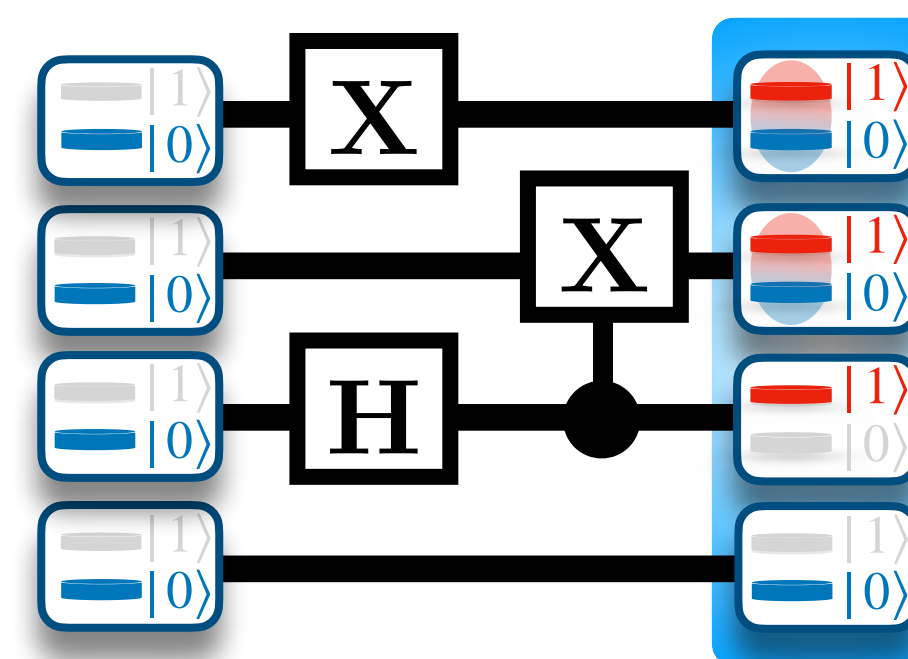
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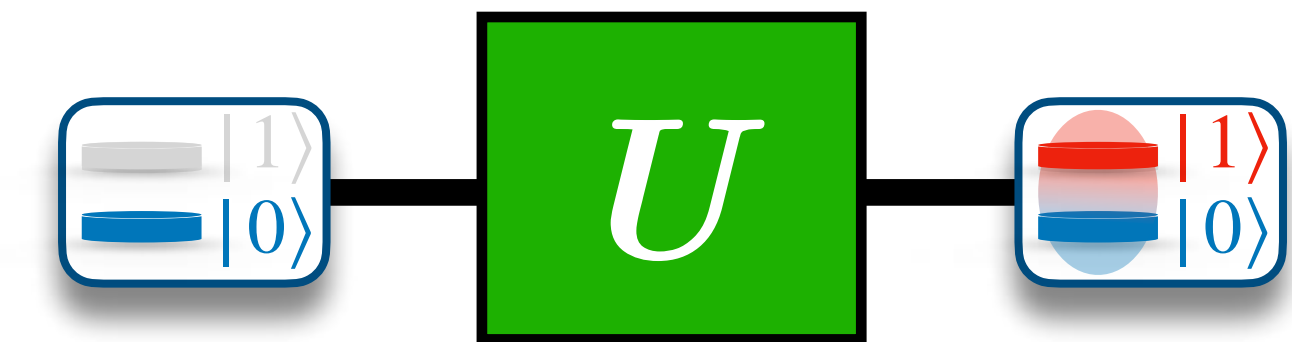


$$|Q\rangle = c_0|0\rangle + c_1|1\rangle$$

Quantum Circuit



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**QUESTION**

Why a Quantum computer is more powerful than a classical one ?

# I) Introduction to quantum computing

***ANSWER***

How to encode information in Qubits vs. Bits

# I) Introduction to quantum computing

**ANSWER**

How to encode information in Qubits vs. Bits

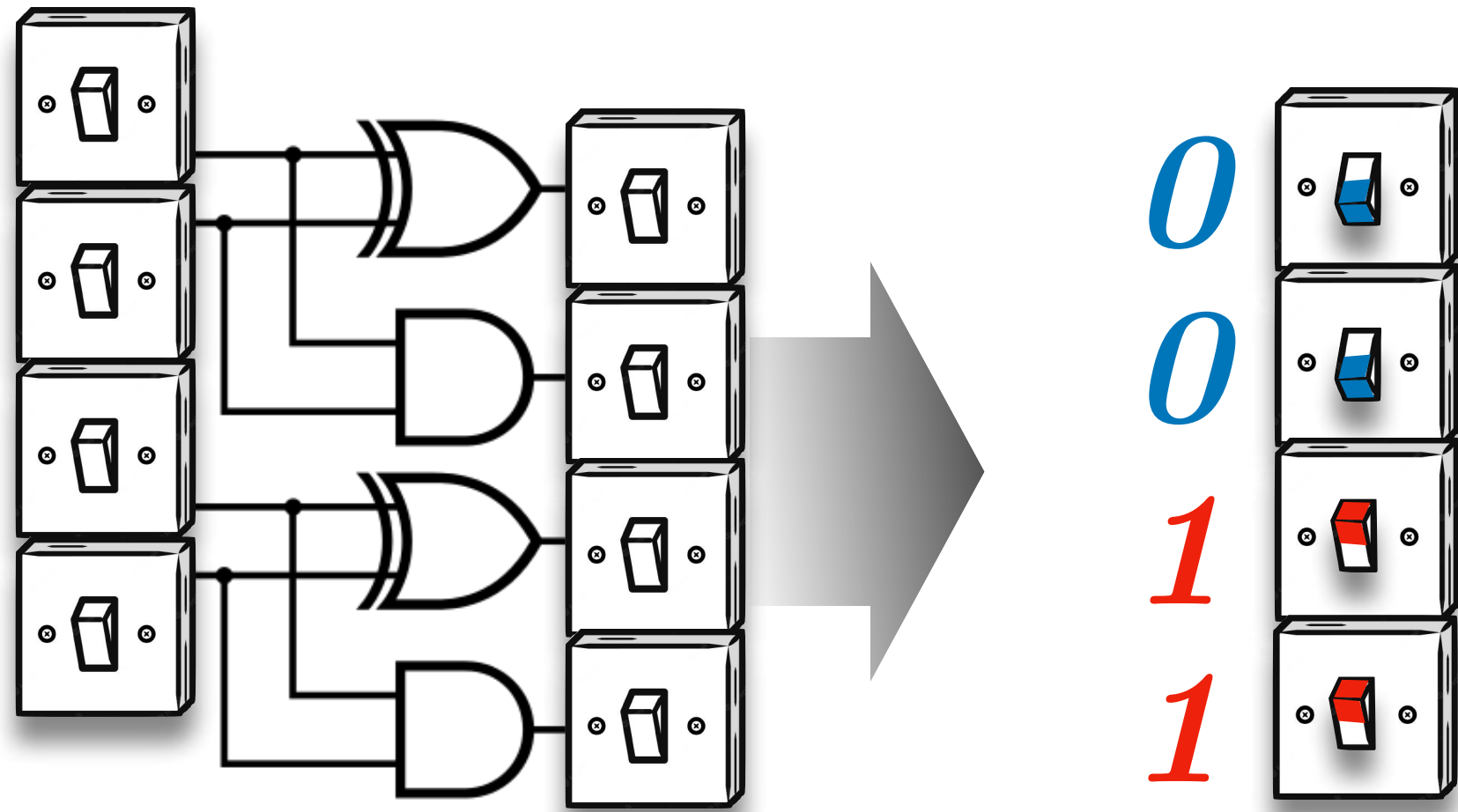
$2^N$  *Bitstrings*  
accessible from  
 $N$  (qu)bits

# I) Introduction to quantum computing

**ANSWER**

How to encode information in Qubits vs. Bits

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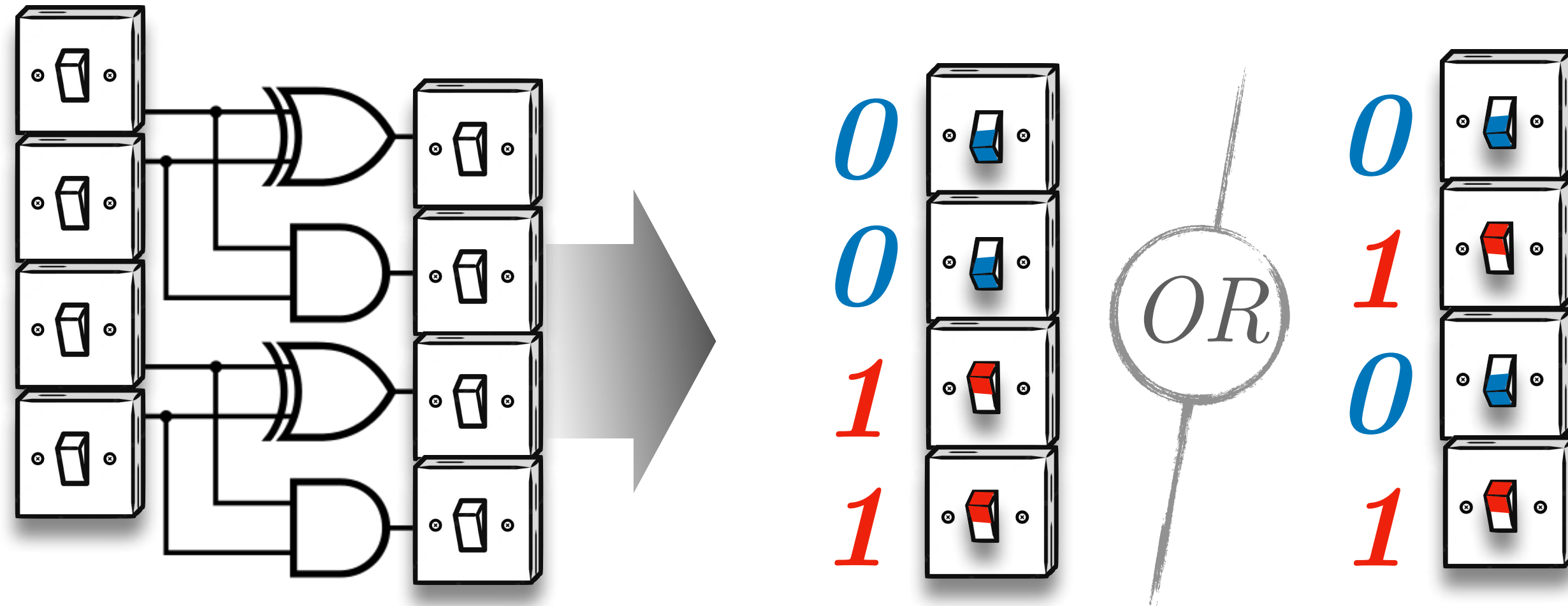


# I) Introduction to quantum computing

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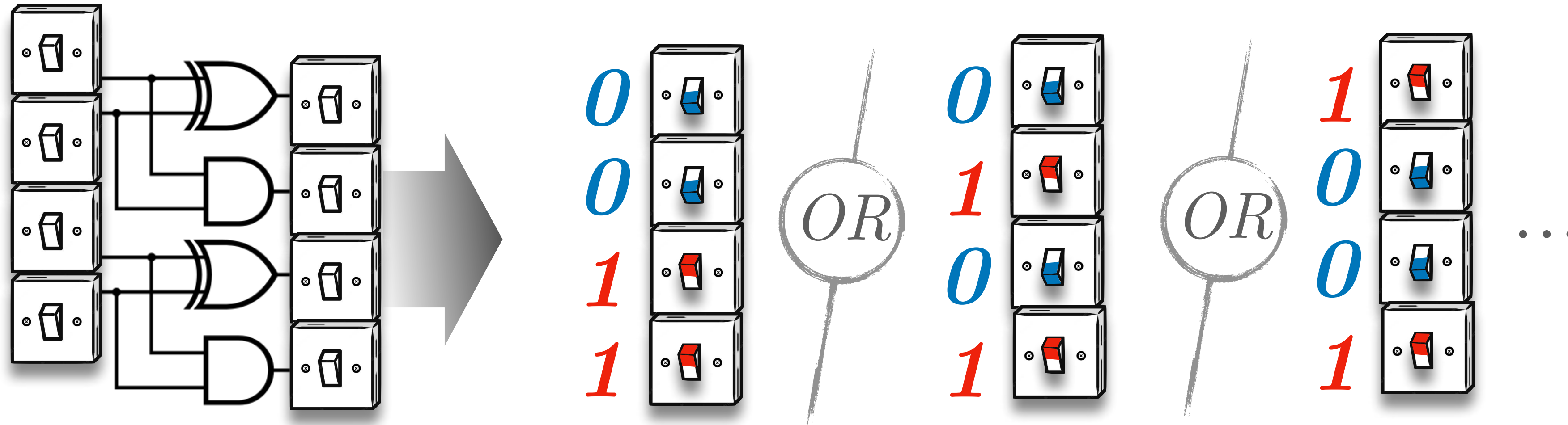


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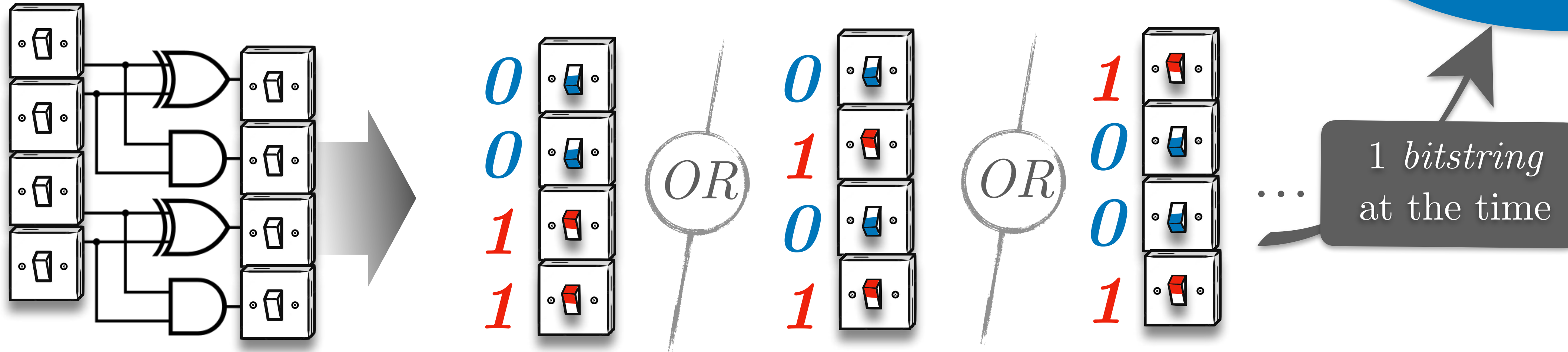


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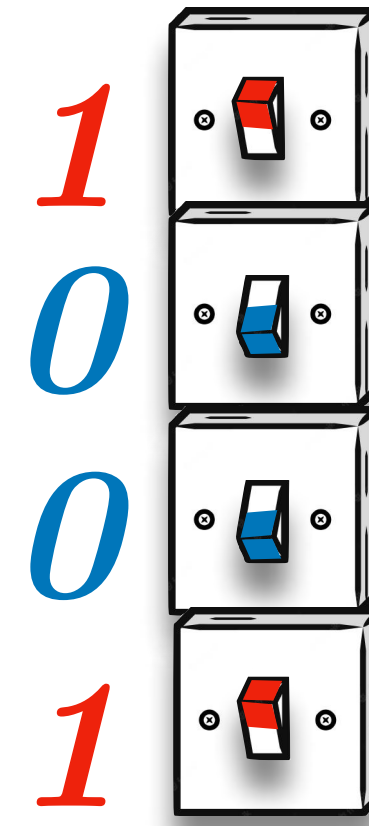
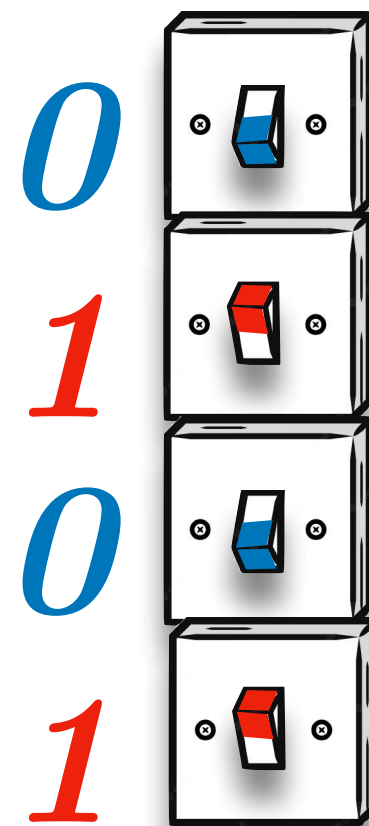
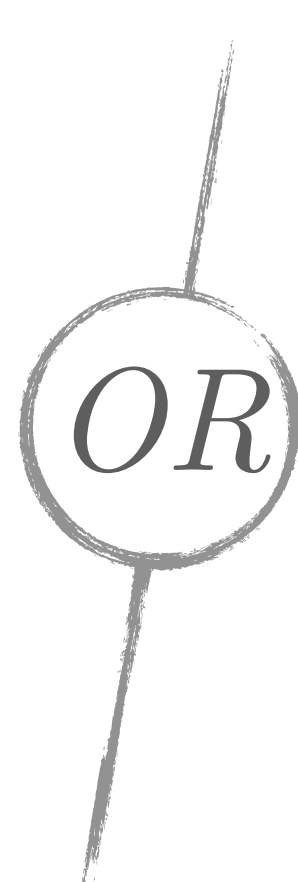
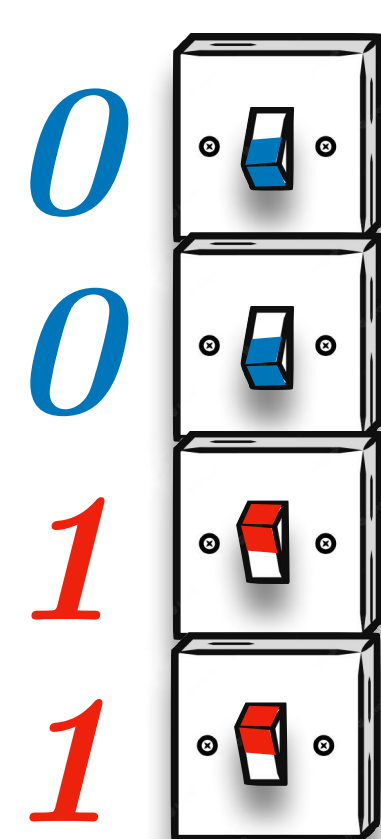
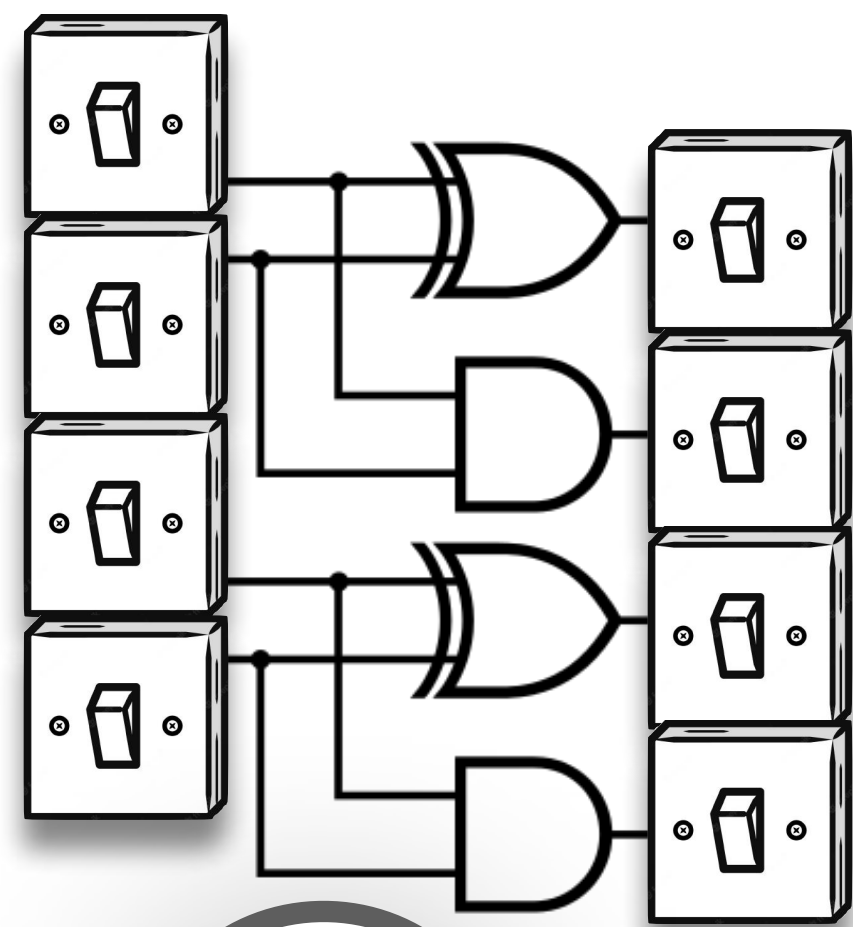


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**ANSWER**

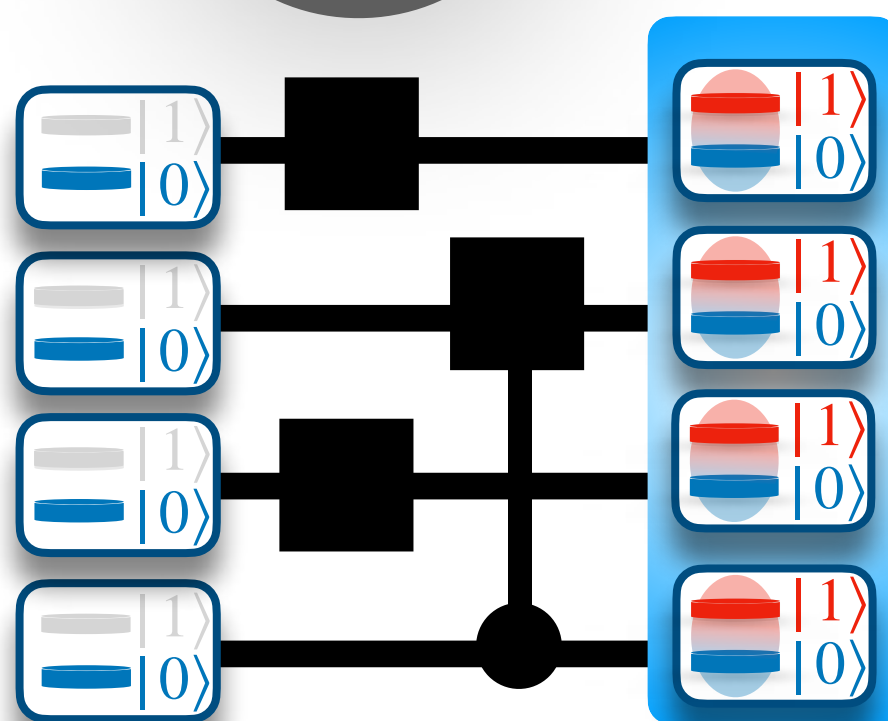
How to encode information in Qubits vs. Bits

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...  
1 bitstring  
at the time

VS.

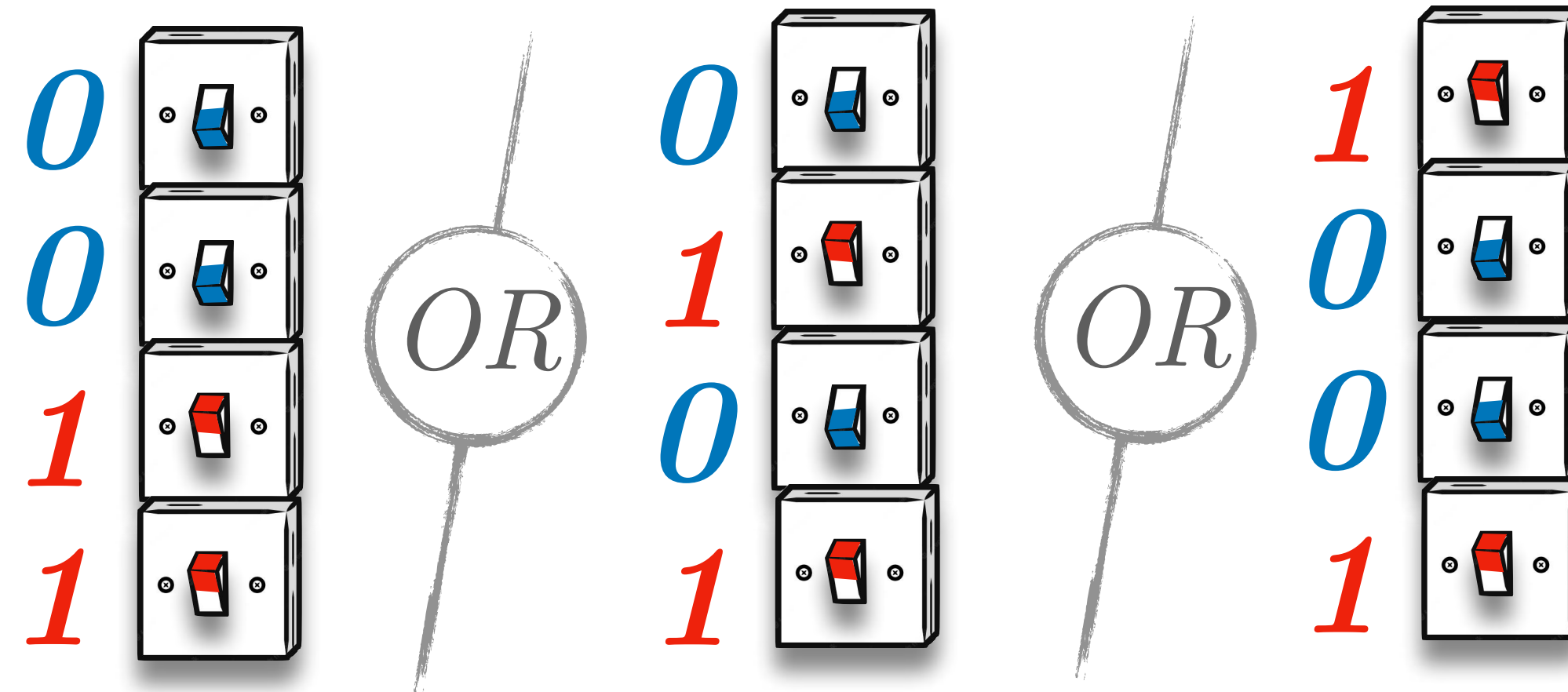
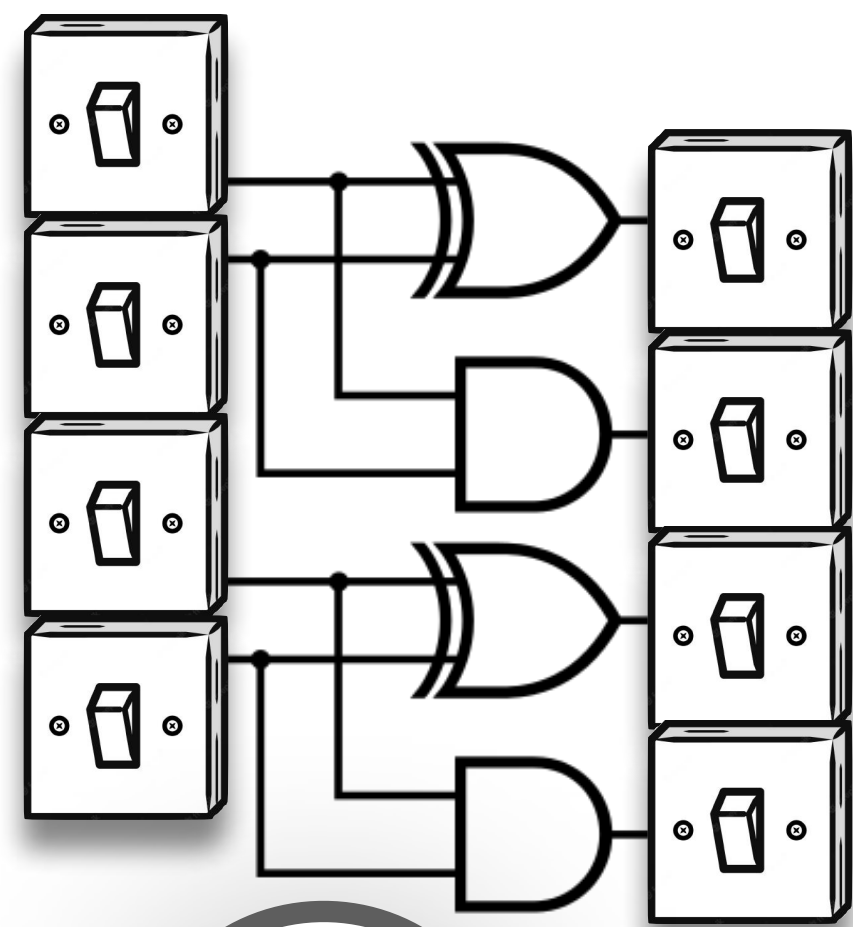


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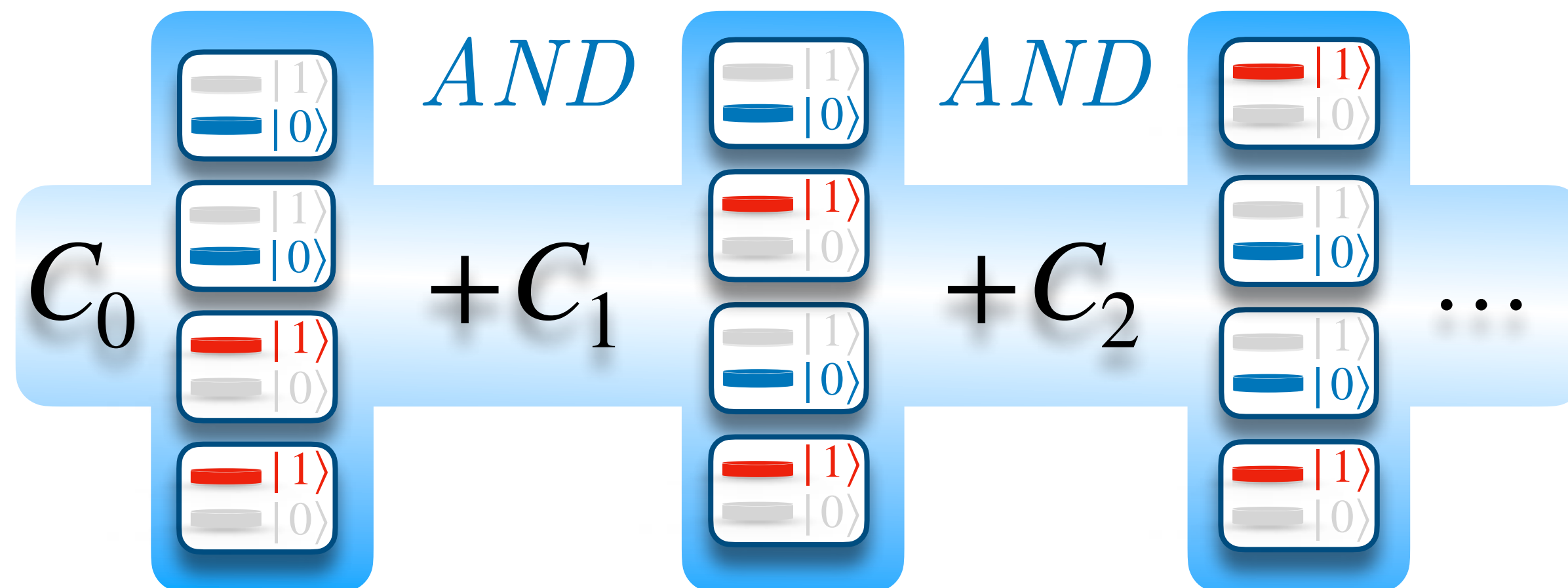
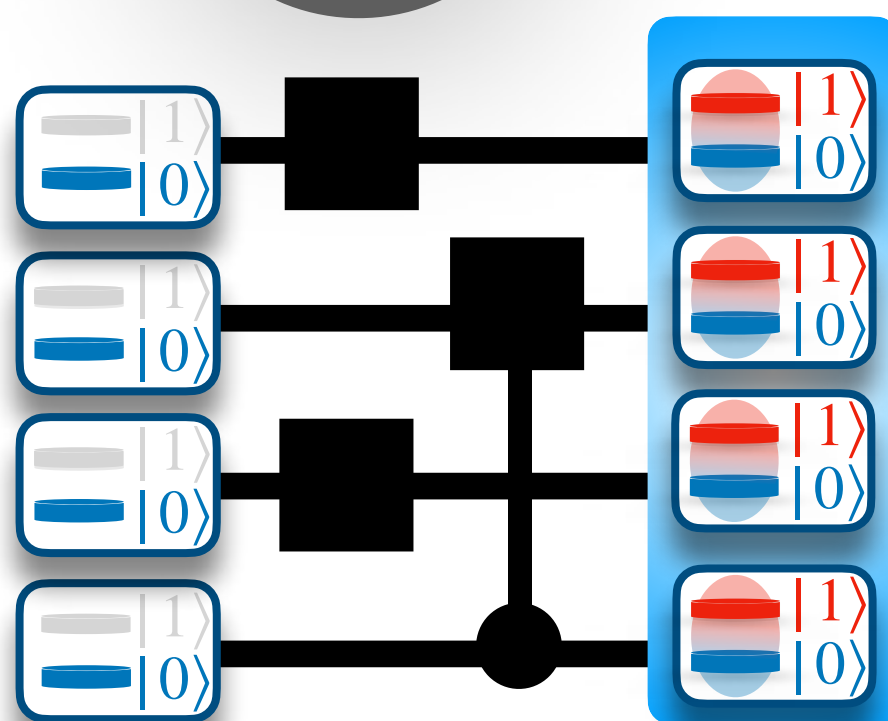
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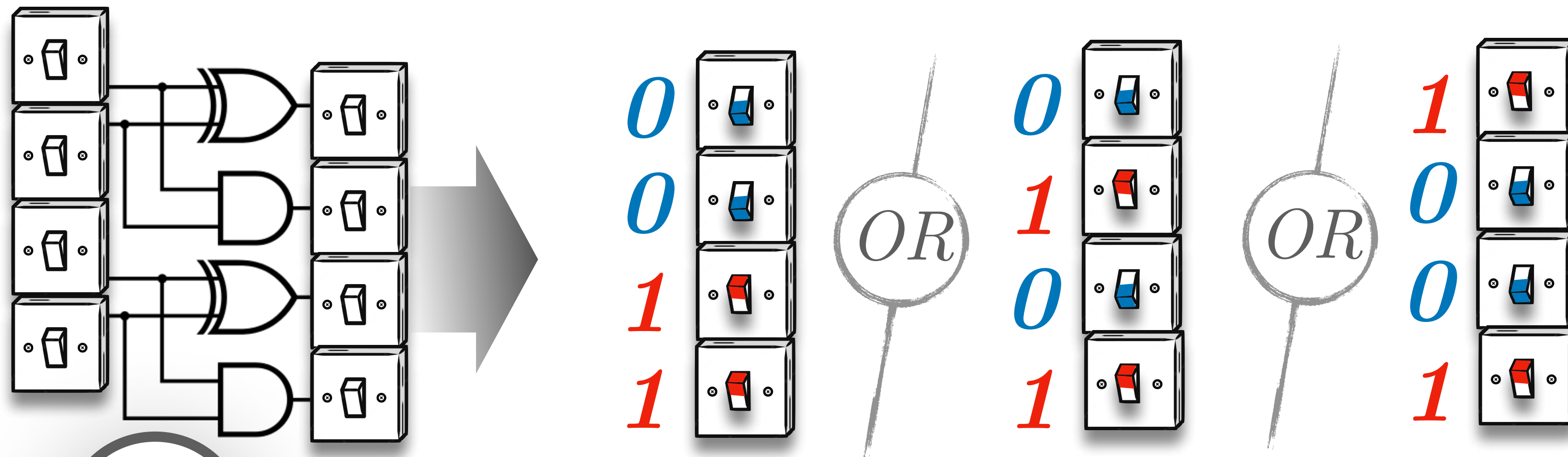


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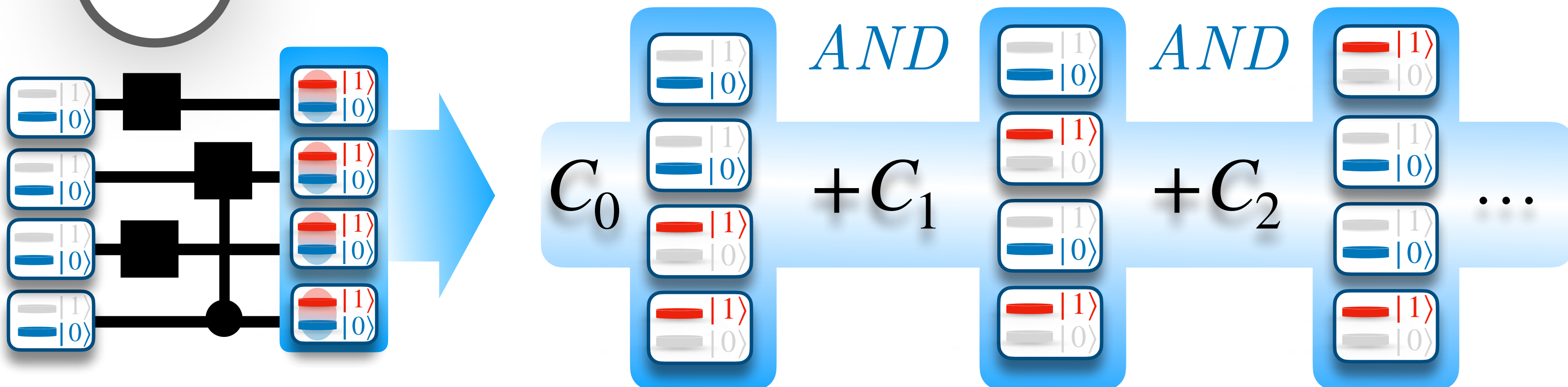
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$2^N$  Bitstrings accessible from  $N$  (qu)bits



1 bitstring at the time

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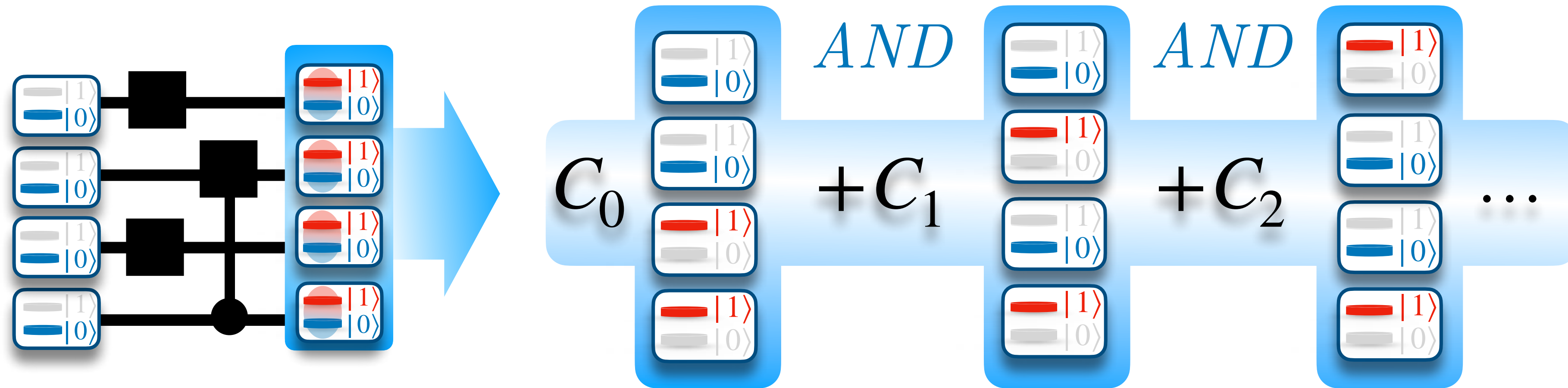


$2^N$  bitstrings simultaneously

$|\Psi\rangle$

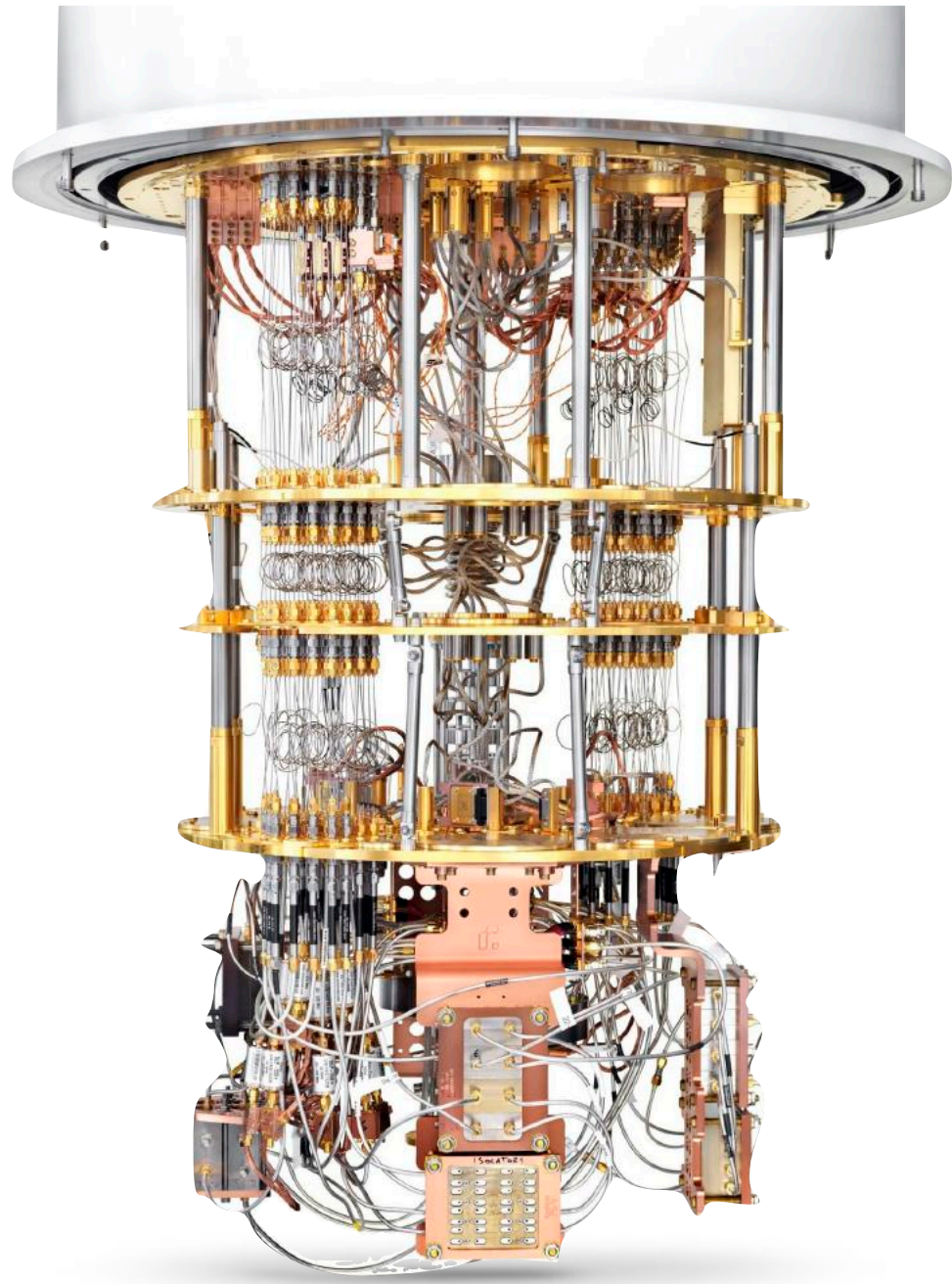
# I) Introduction to quantum computing

## Example of Full Quantum Superposition

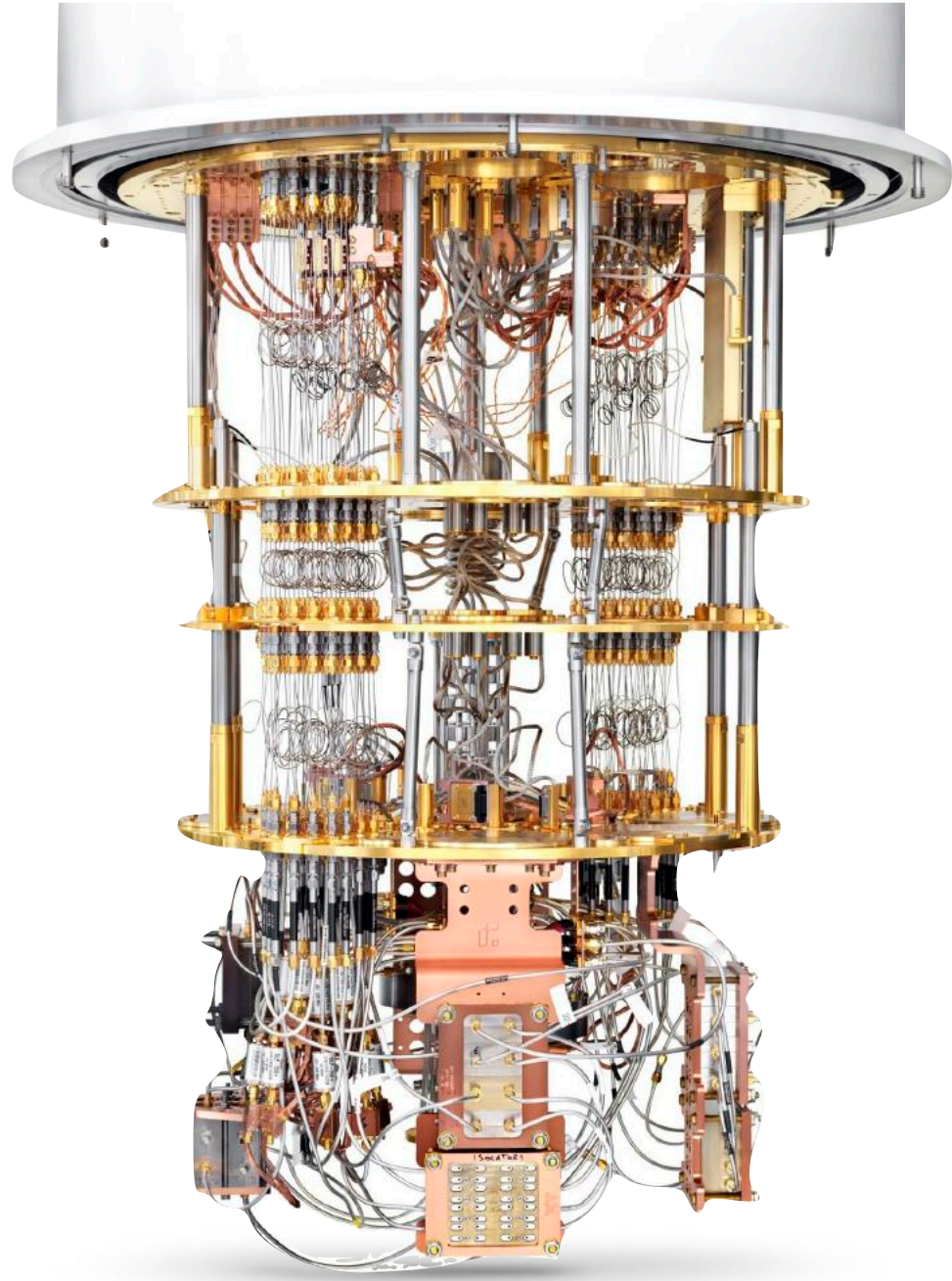


*Let's build a quantum circuit !*

# I) Introduction to quantum computing

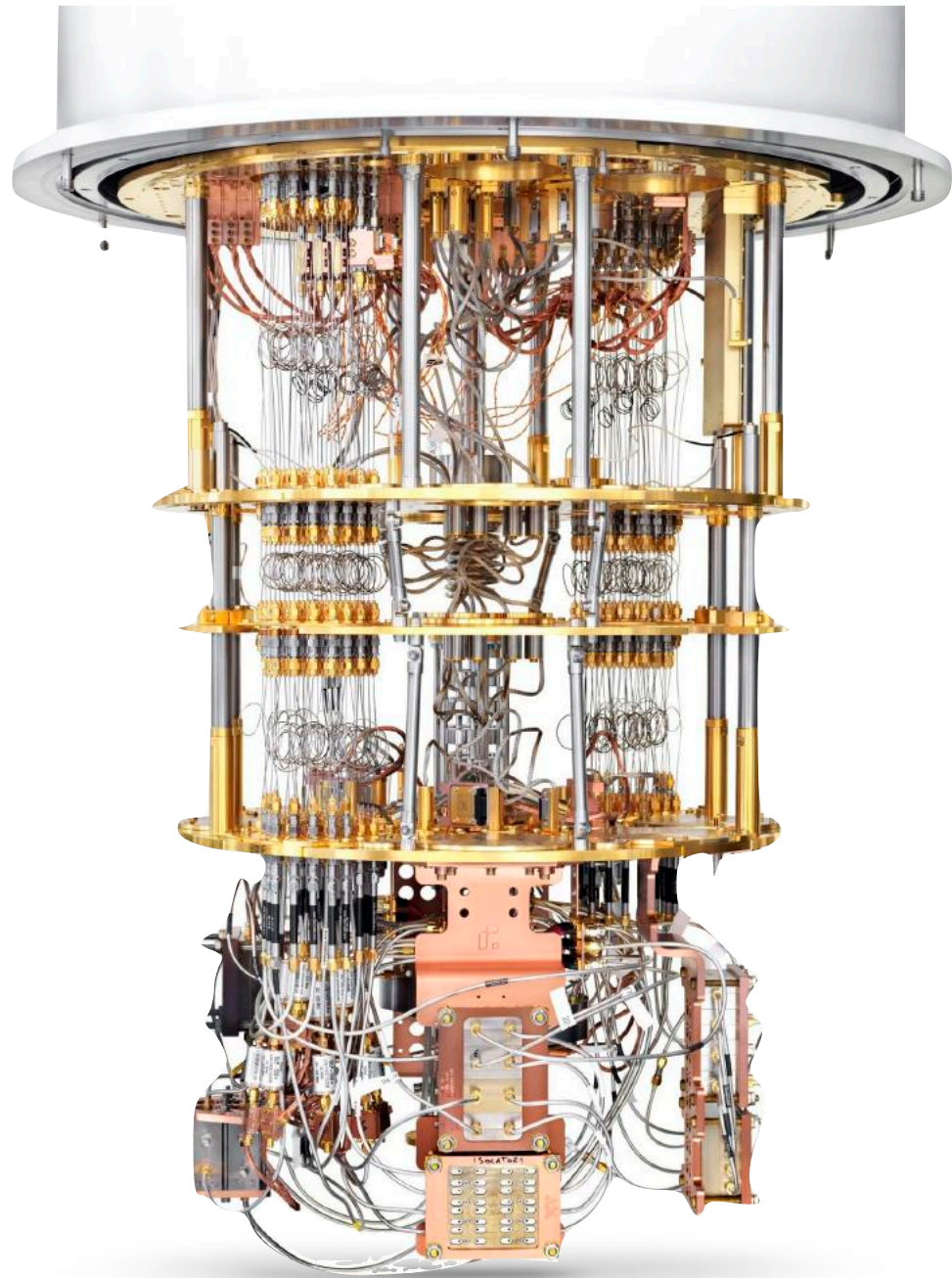


# I) Introduction to quantum computing



Emerging quantum computers are “NISQ” devices.  
(NISQ : Noisy Intermediate-Scale Quantum)

# I) Introduction to quantum computing



Emerging quantum computers are “NISQ” devices.

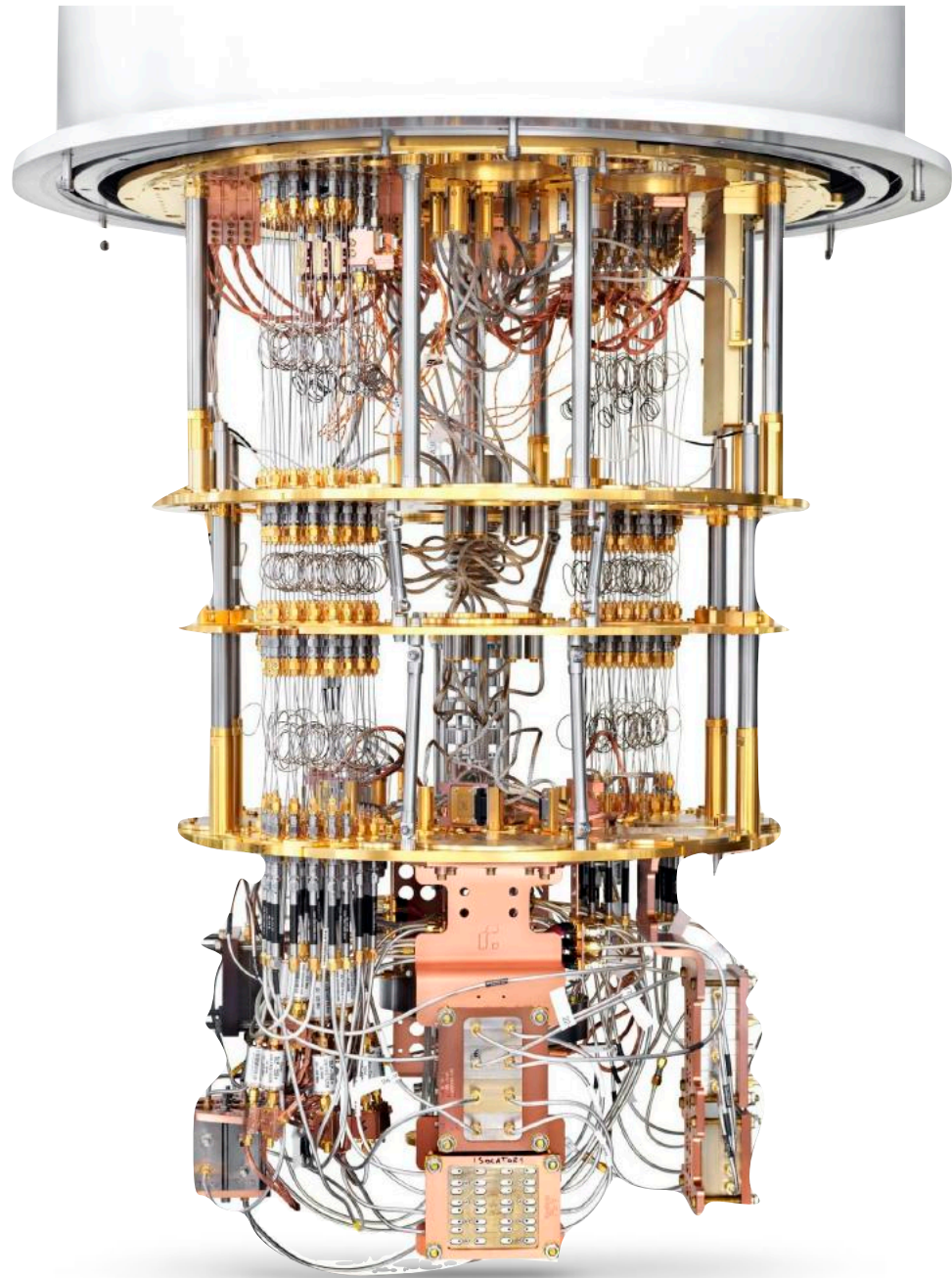
(NISQ : **Noisy** Intermediate-Scale Quantum)

Quantum decoherence

(qubits = open quantum system).



# I) Introduction to quantum computing



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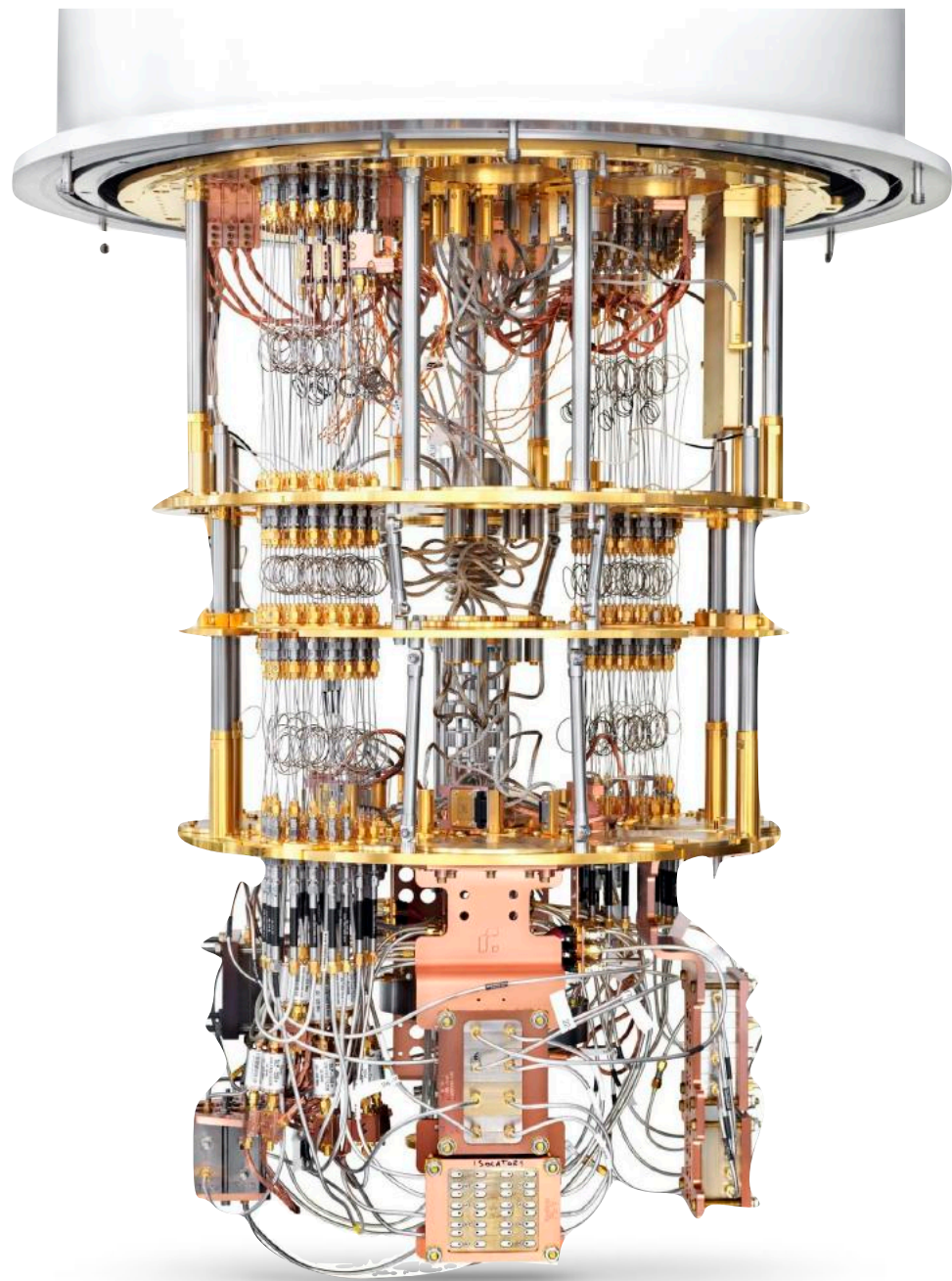
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Only a few qubits accessible

$N_{qubits} \sim 10$

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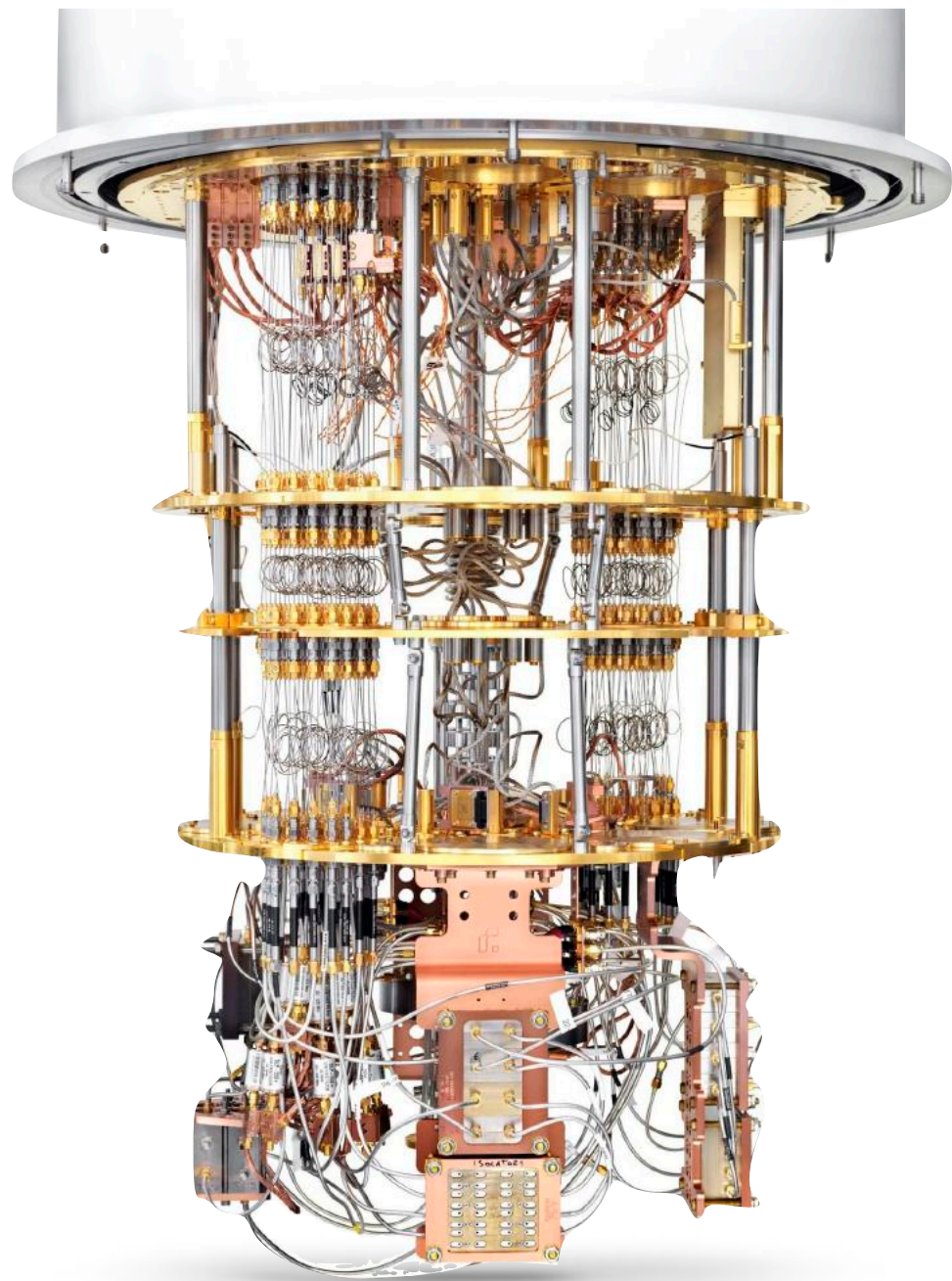
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*NISQ algorithms*

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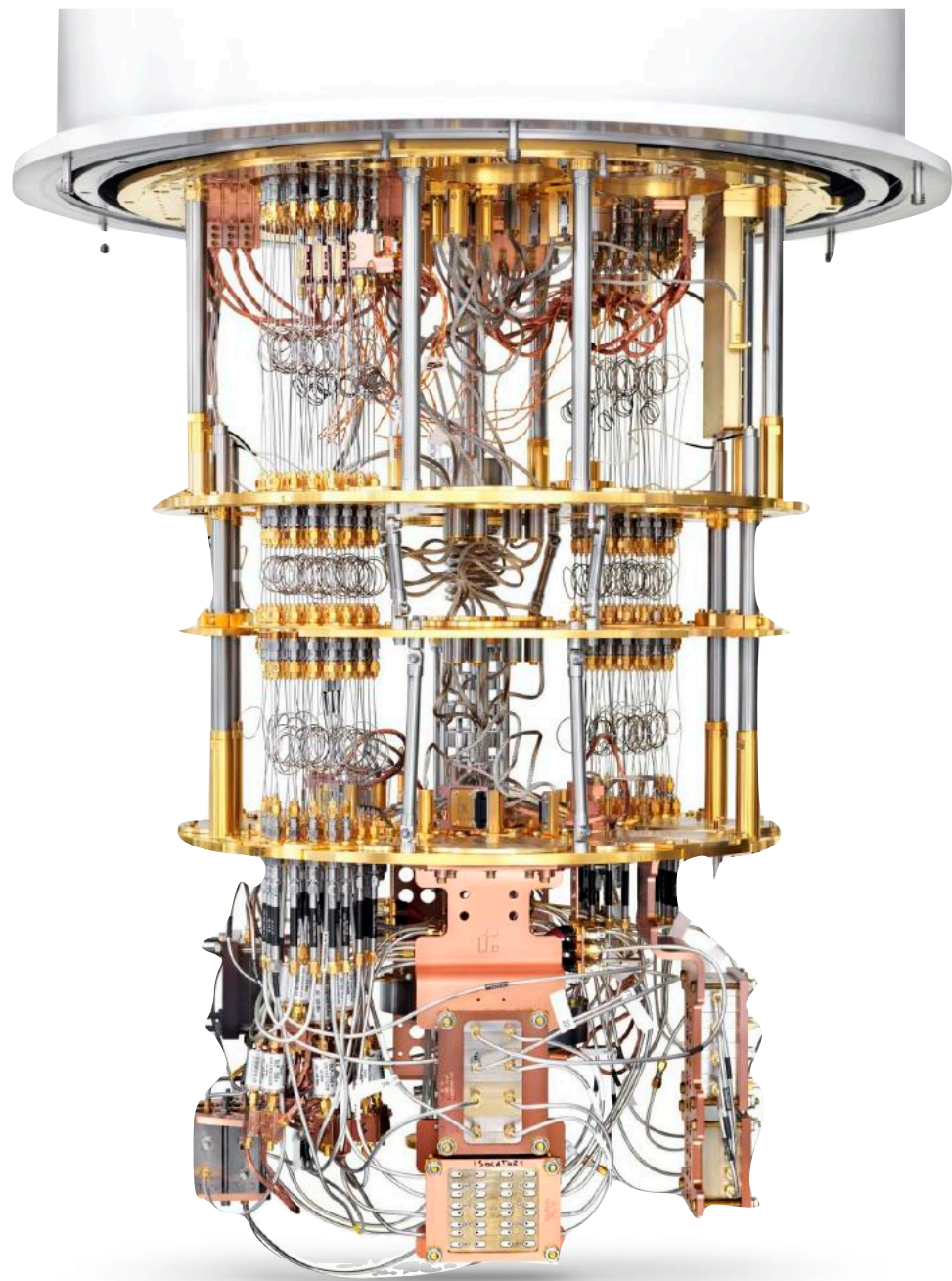
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## *NISQ algorithms*

- *Exponentially* fewer resources to store information

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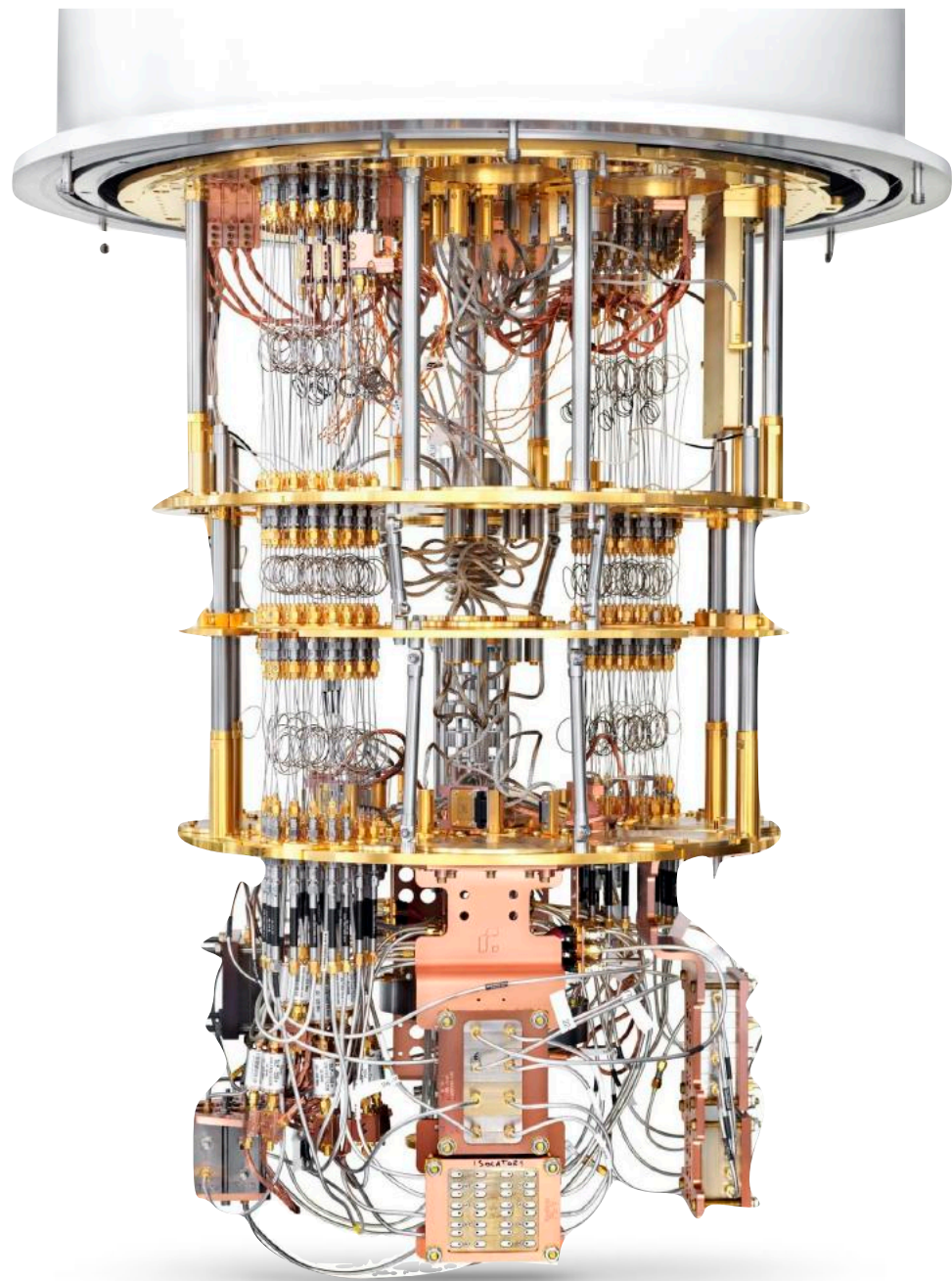
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## *NISQ algorithms*

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- Based on a few qubits and quantum gates.

# I) Introduction to quantum computing



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Only a few qubits accessible

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## *NISQ algorithms*

- *Exponentially* fewer resources to store information
- Based on a few qubits and quantum gates.
- Pretty resistant to the noise effects.

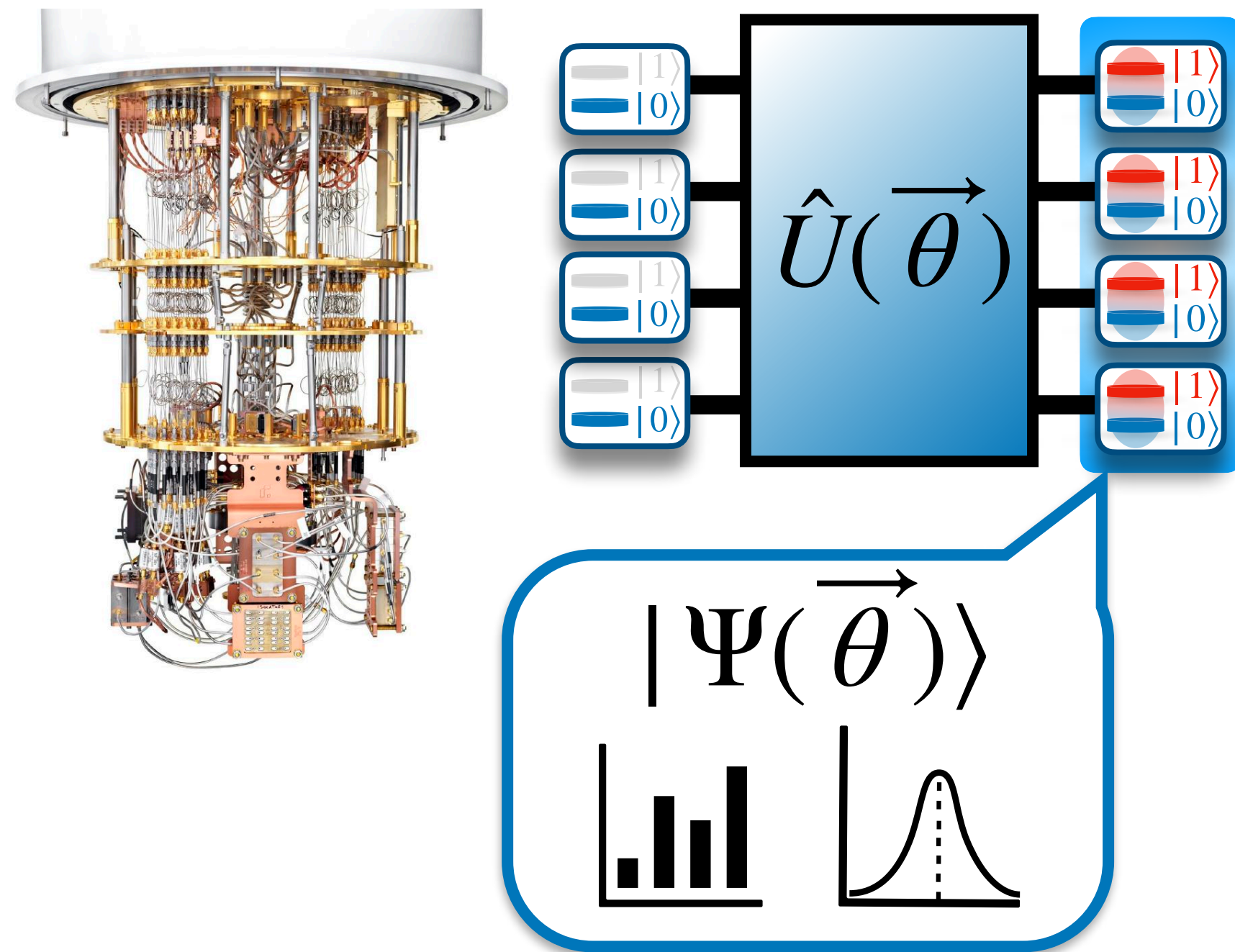
# I) Introduction to quantum computing

*NISQ algorithm:* Hybrid Quantum/Classical methods

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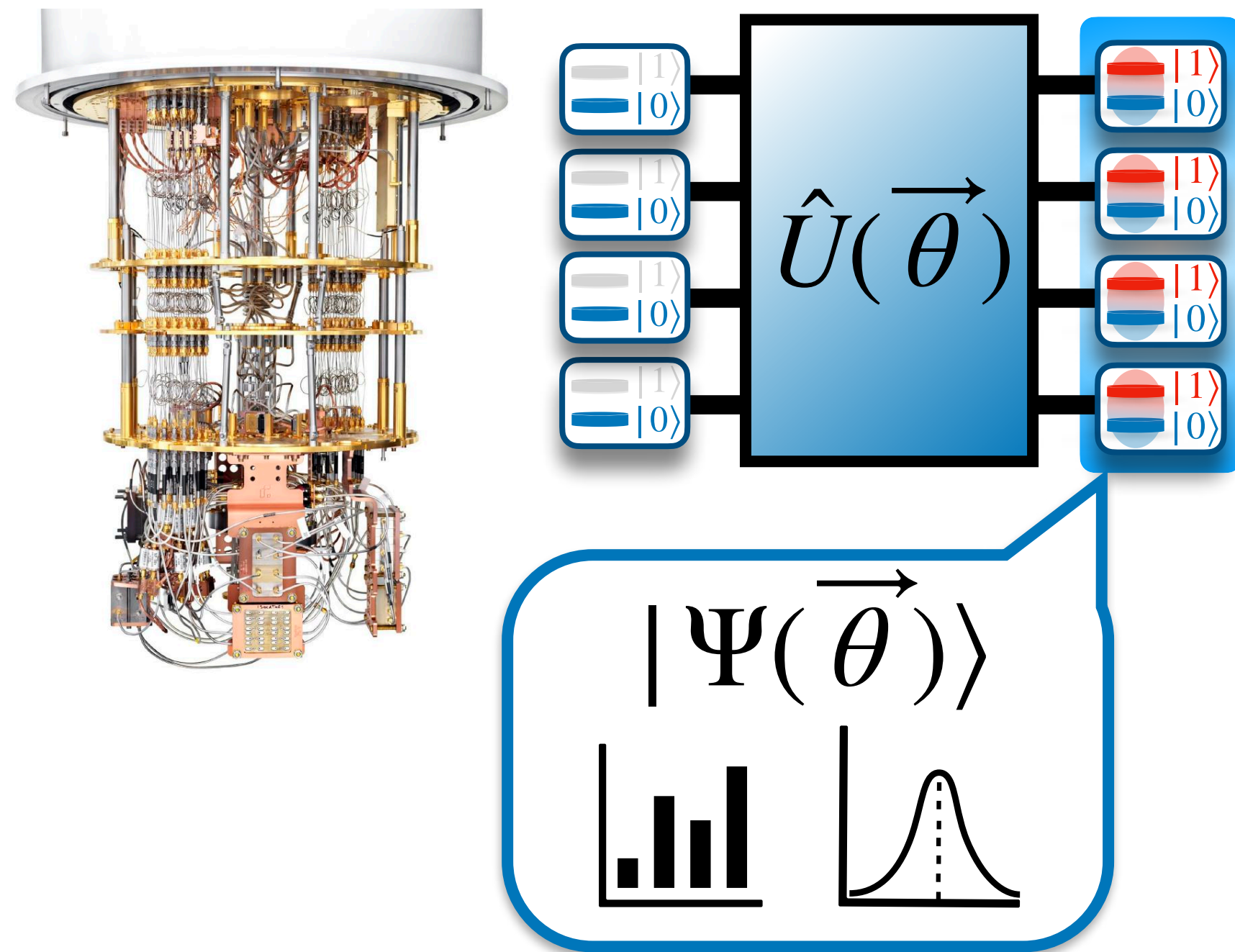
*Quantum Computer*



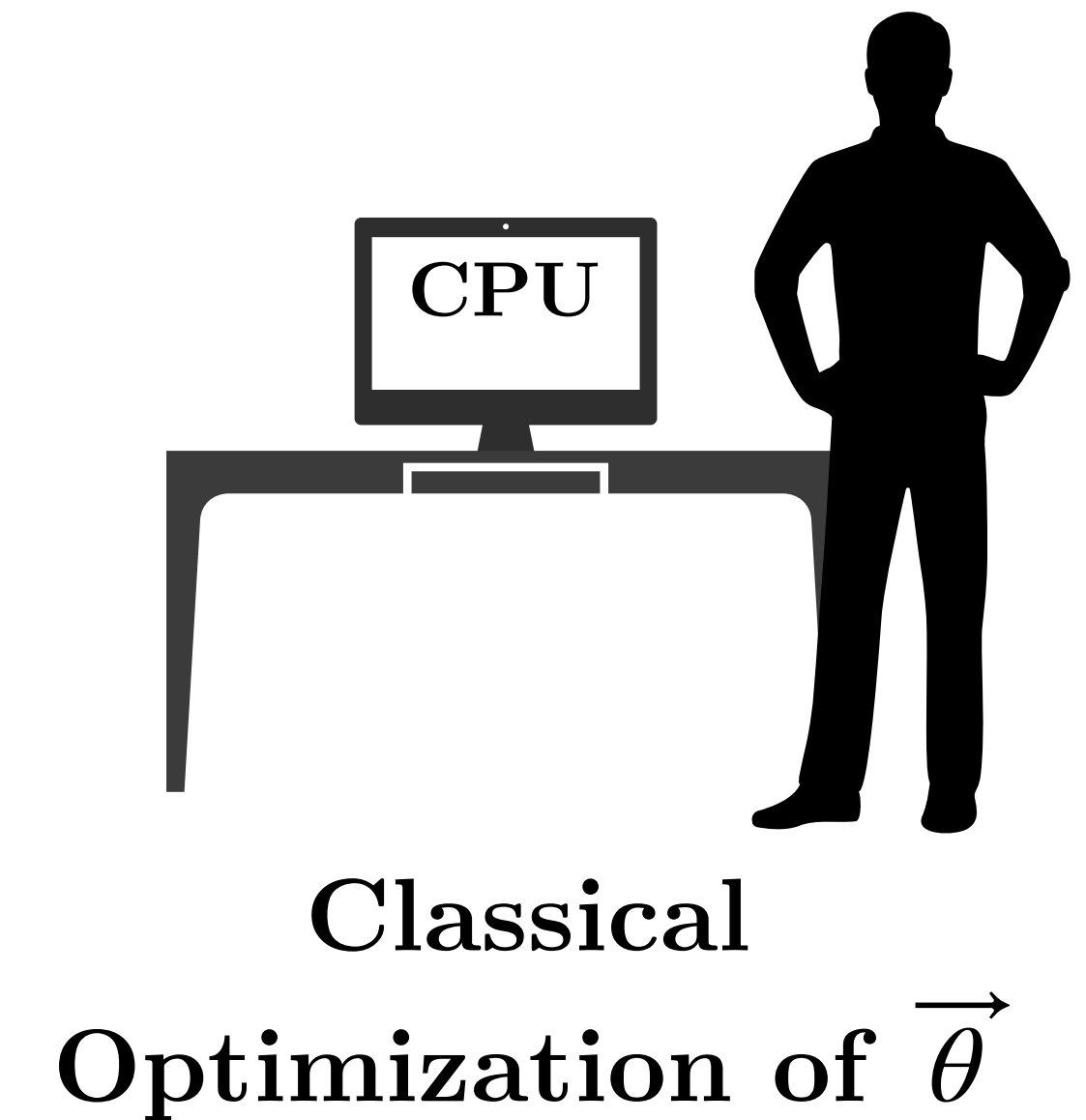
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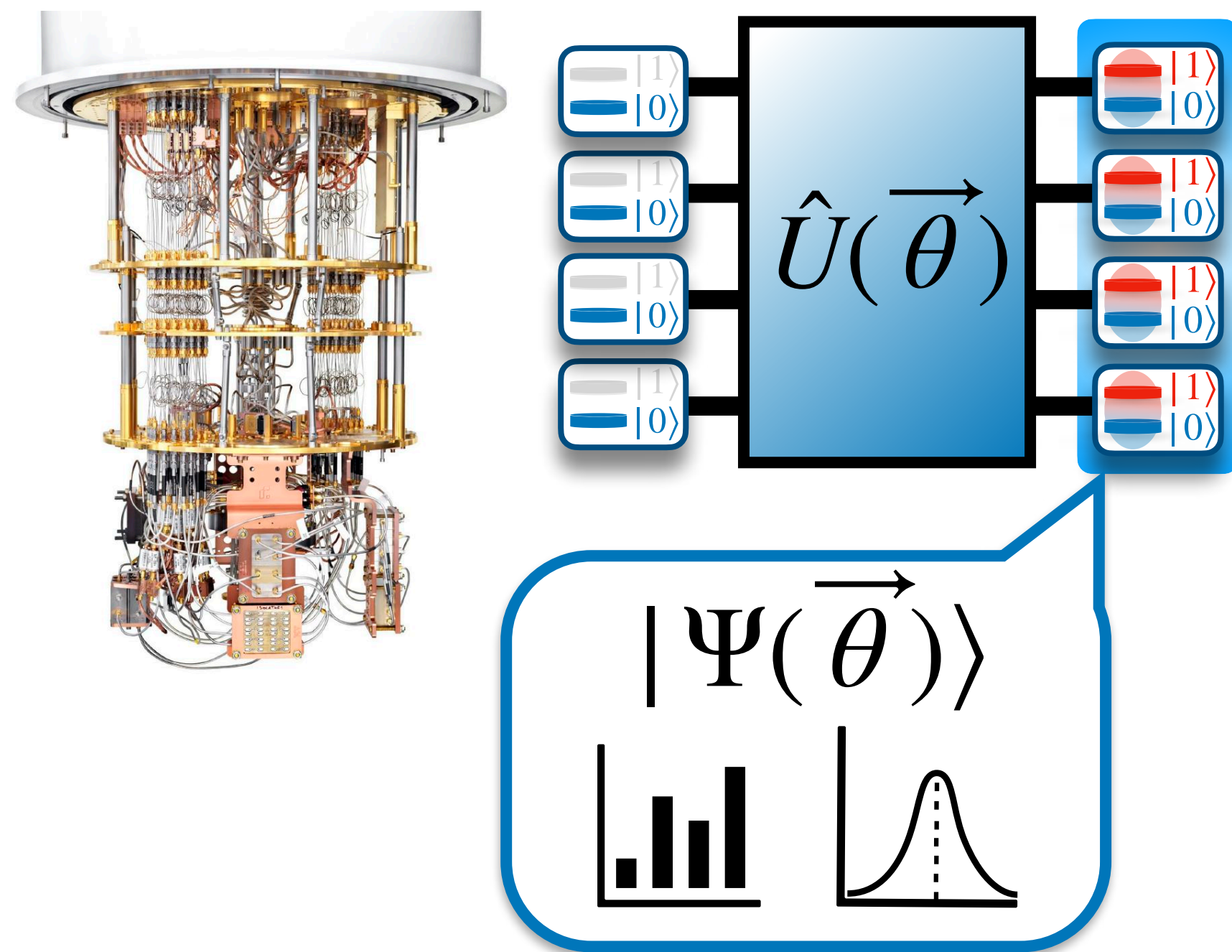




# I) Introduction to quantum computing

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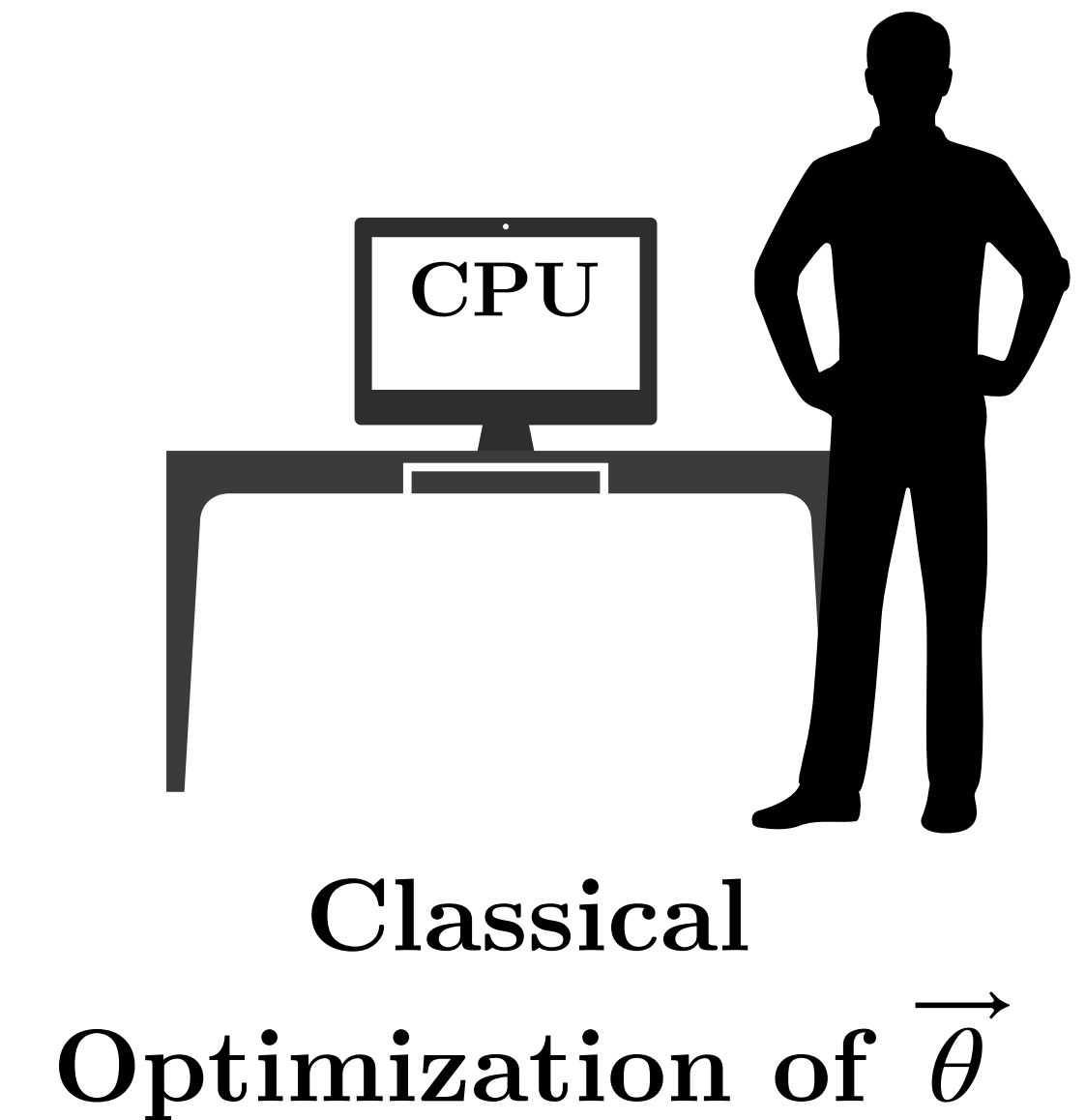
*Quantum Computer*



*Classical Computer*

Measure

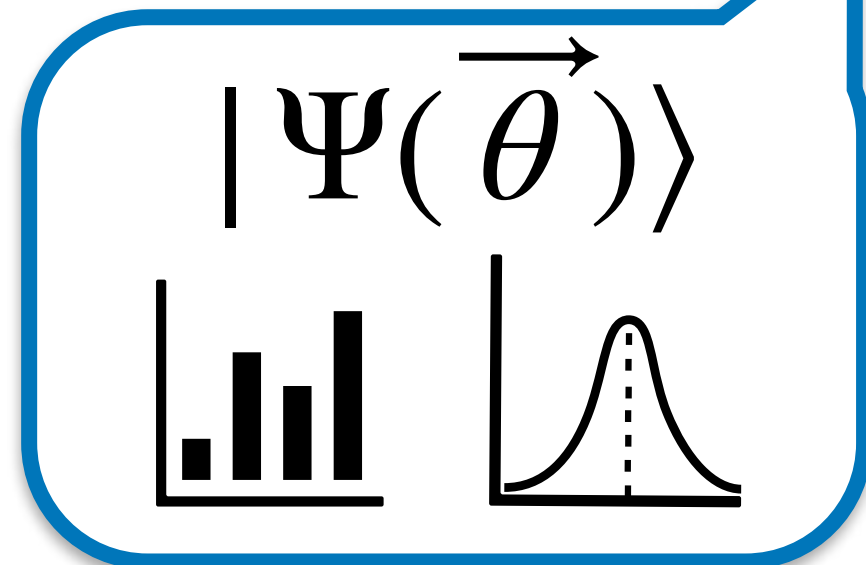
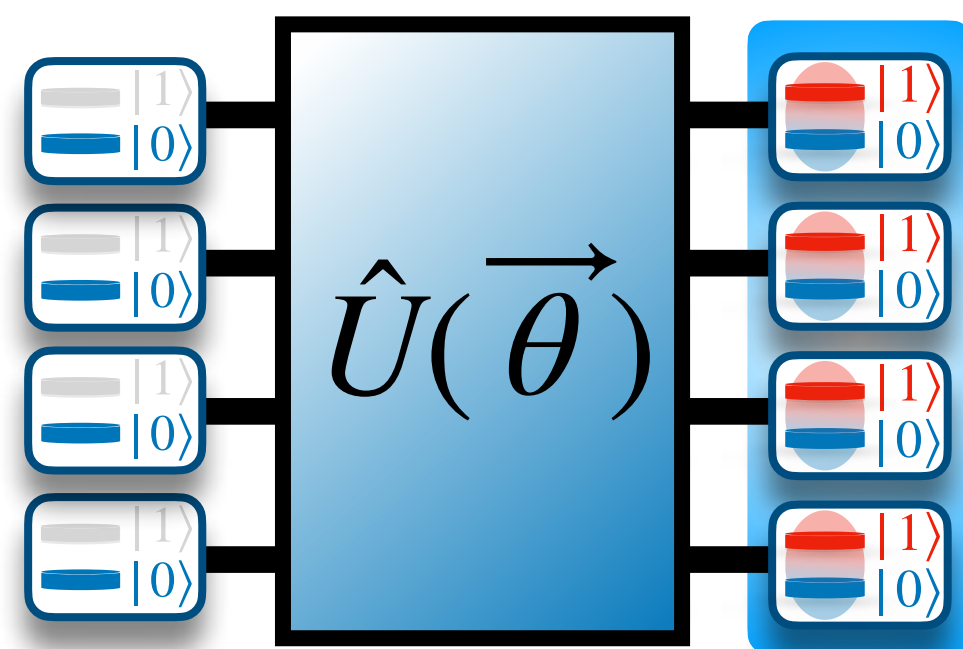
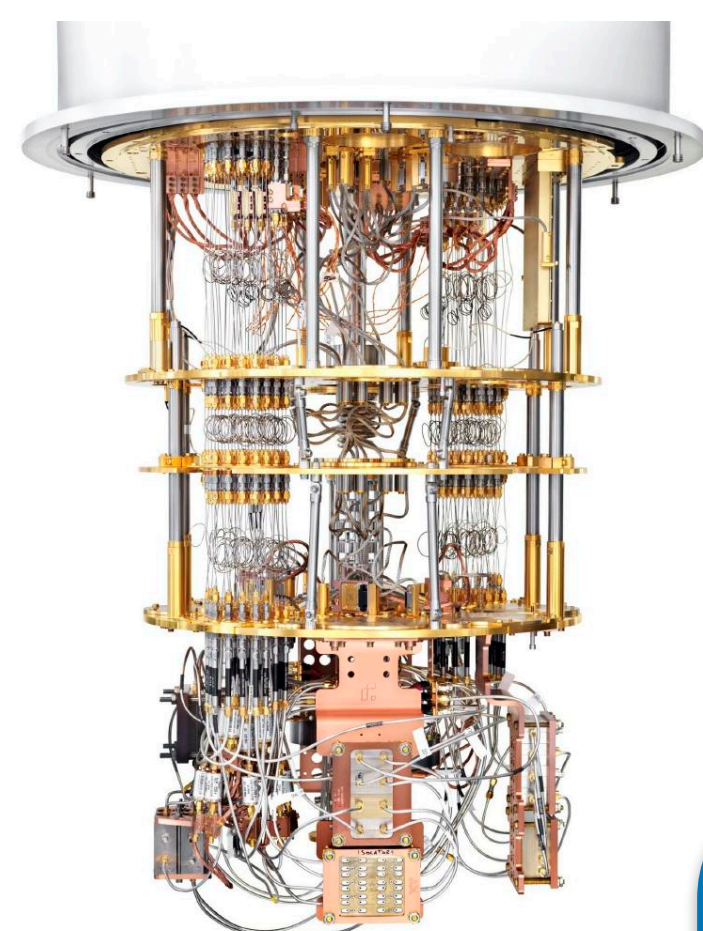
$$\langle \hat{O}_{\Psi(\vec{\theta})} \rangle = \langle \Psi(\vec{\theta}) | \hat{O} | \Psi(\vec{\theta}) \rangle$$



# I) Introduction to quantum computing

*NISQ algorithm: Hybrid Quantum/Classical methods*

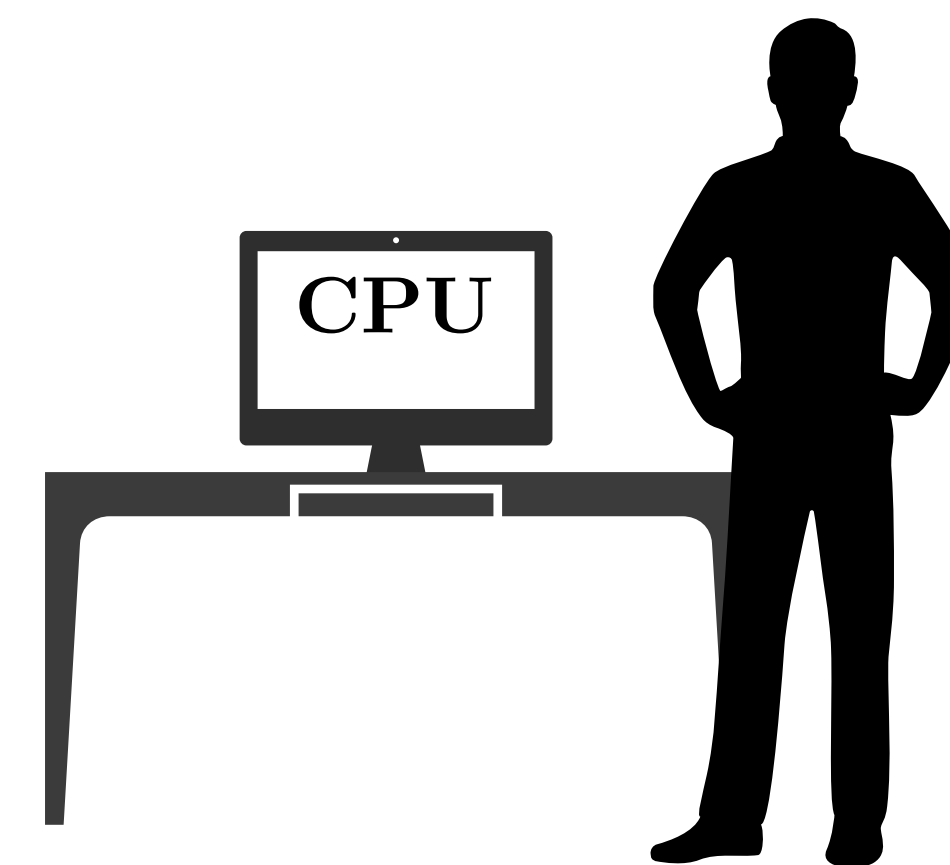
*Quantum Computer*



Circuit parameters  $\vec{\theta}$   
to generate a new  $|\Psi(\vec{\theta})\rangle$

Measure  
 $\langle \hat{O}_{\Psi(\vec{\theta})} \rangle = \langle \Psi(\vec{\theta}) | \hat{O} | \Psi(\vec{\theta}) \rangle$

*Classical Computer*

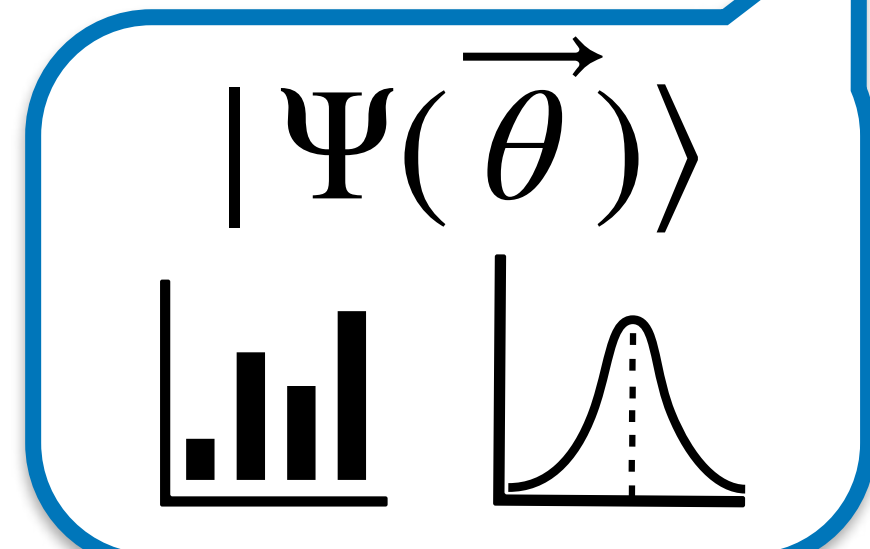
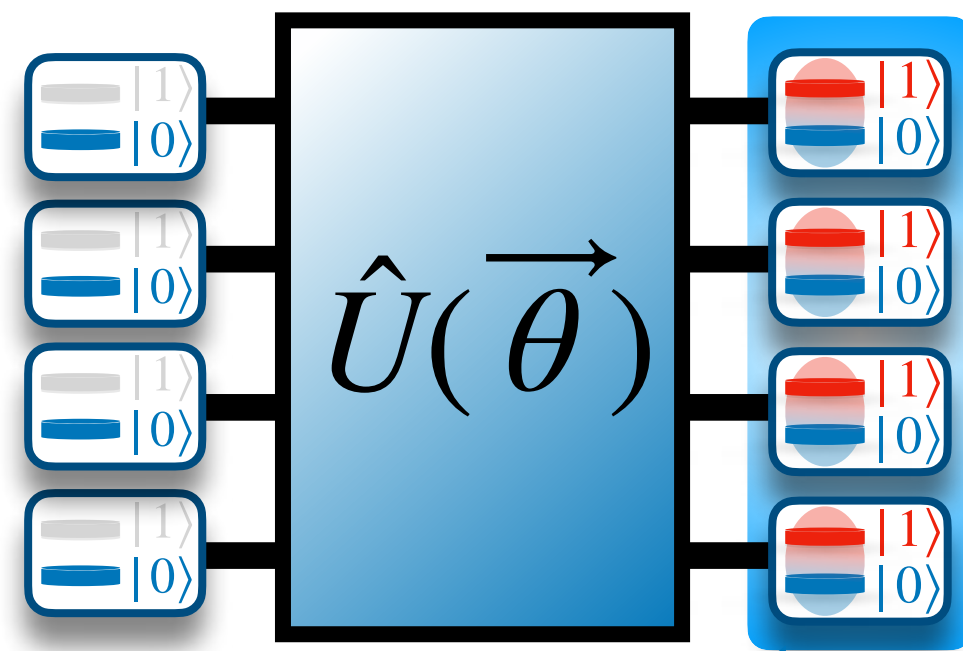
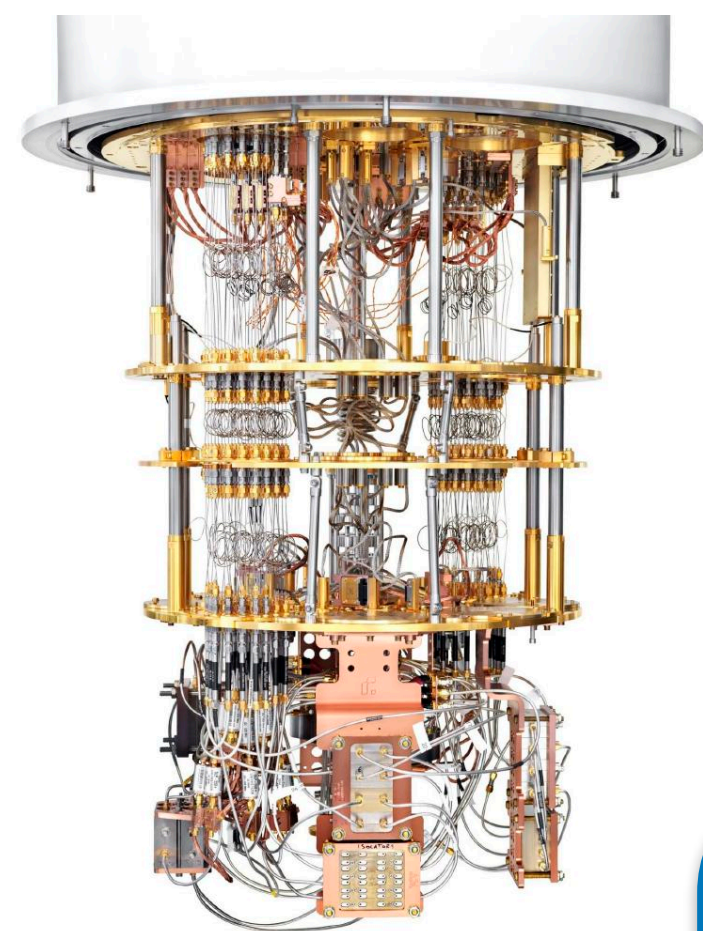


Classical  
Optimization of  $\vec{\theta}$

# I) Introduction to quantum computing

*NISQ algorithm:* Hybrid Quantum/Classical methods

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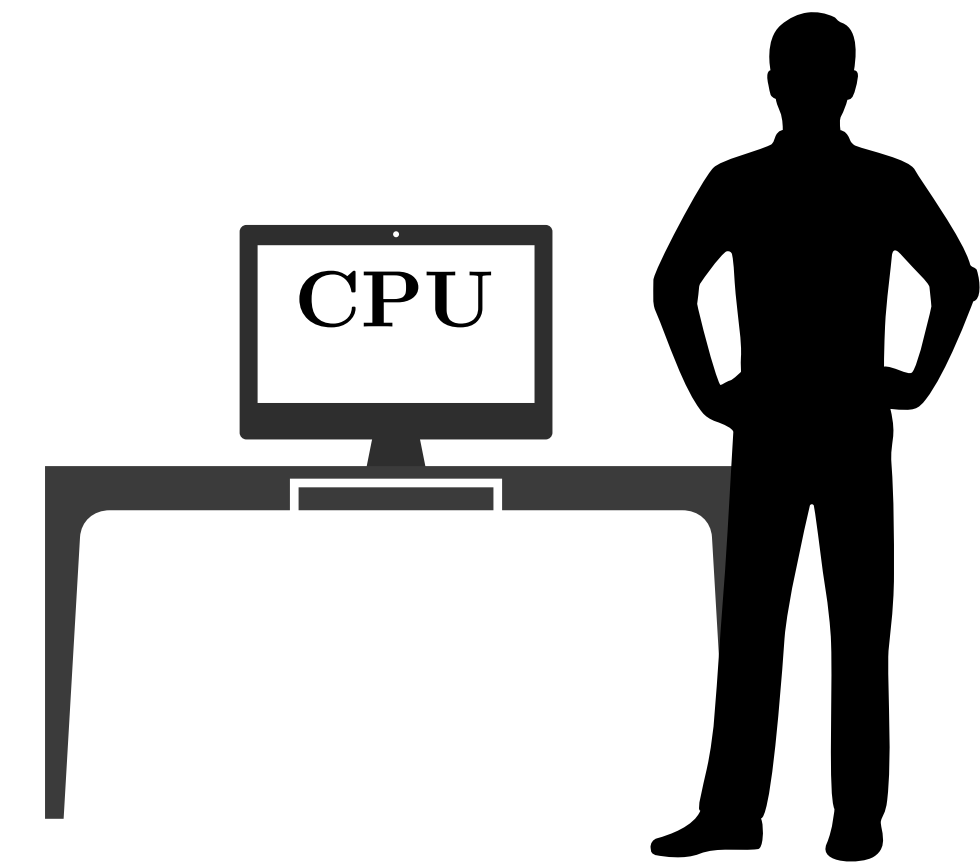
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**Applications**

- Quantum machine learning
- Quantum Chemistry

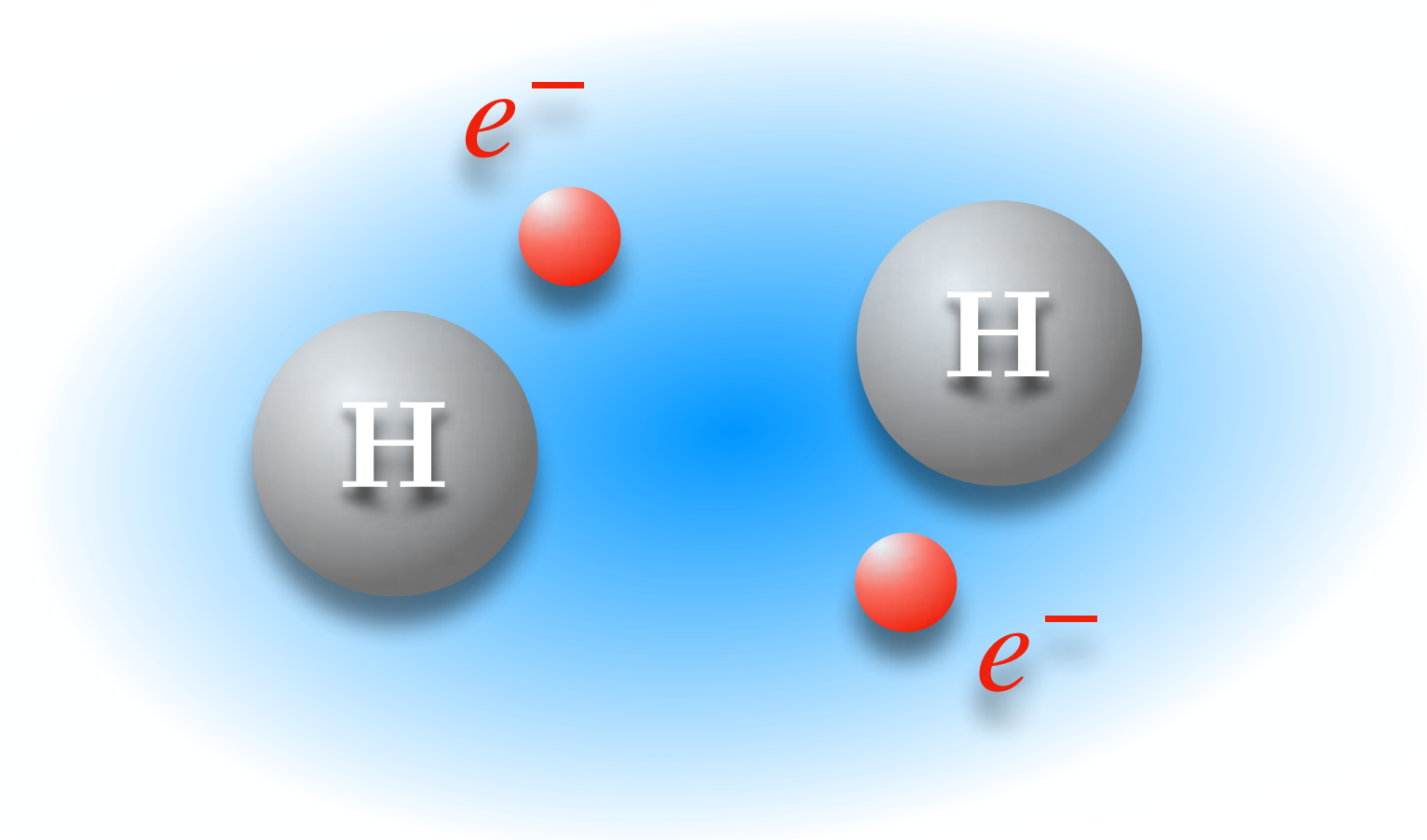
*Classical Computer*



**Classical  
Optimization of  $\vec{\theta}$**

## II) From quantum computing to chemistry

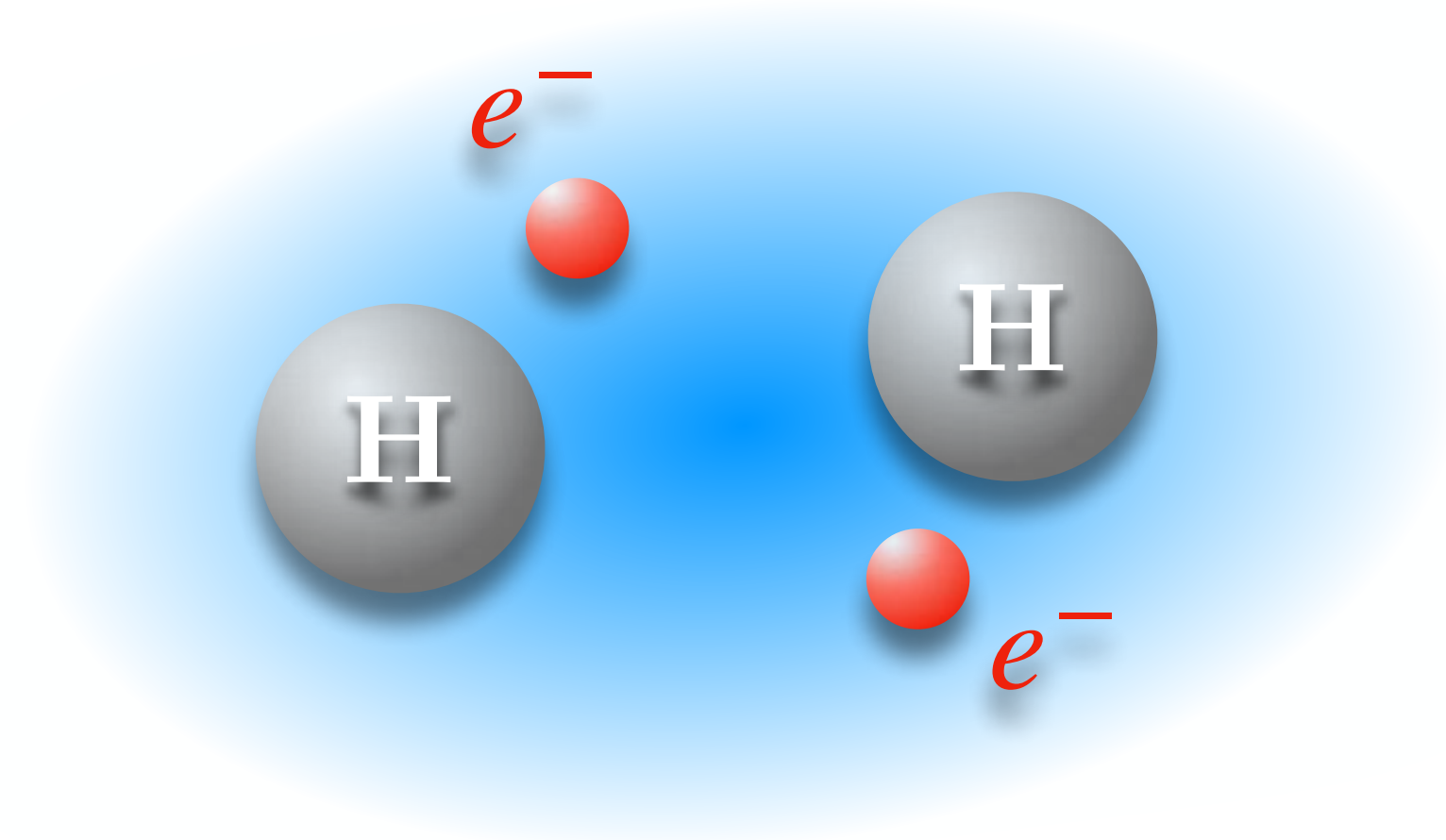
## II) From quantum computing to chemistry



**Electronic structure Hamiltonian**  
(Born-Oppenheimer approximation)

$$H = -\frac{1}{2} \sum_{i=1}^{N_e} \nabla_{r_i}^2 - \sum_{i=1}^{N_e} \sum_{A=1}^{N_a} \frac{Z_A}{|r_i - R_A|} + \frac{1}{2} \sum_{i \neq j}^{N_e} \frac{1}{|r_i - r_j|}$$

## II) From quantum computing to chemistry



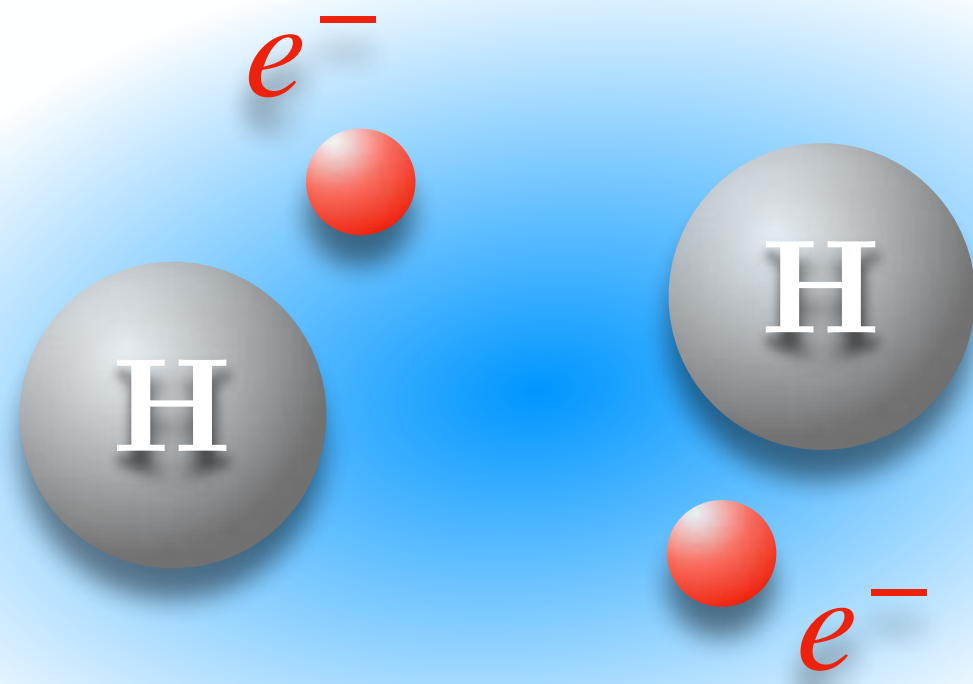
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**Finding  
Ground State** →

$$H |\Psi_0\rangle = E_0 |\Psi_0\rangle$$

## II) From quantum computing to chemistry



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Finding  
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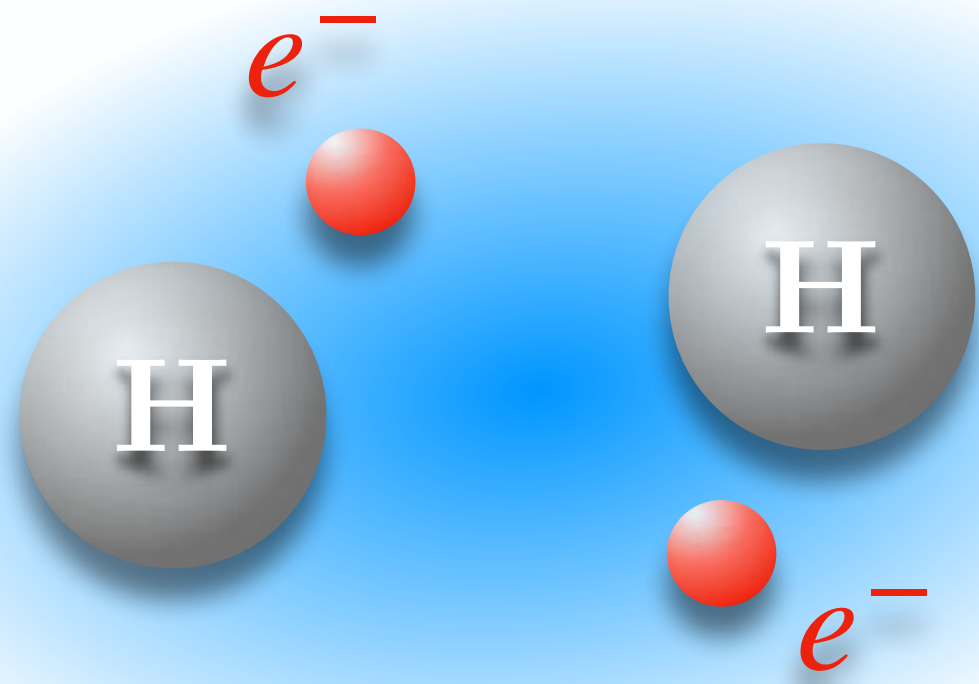
$$H|\Psi_0\rangle = E_0|\Psi_0\rangle$$

**Mean-Field Approach**  
(Hartree-Fock)

Single Configuration  
Approximation

$$|\Psi_0\rangle \approx |\Phi_{HF}\rangle$$

## II) From quantum computing to chemistry



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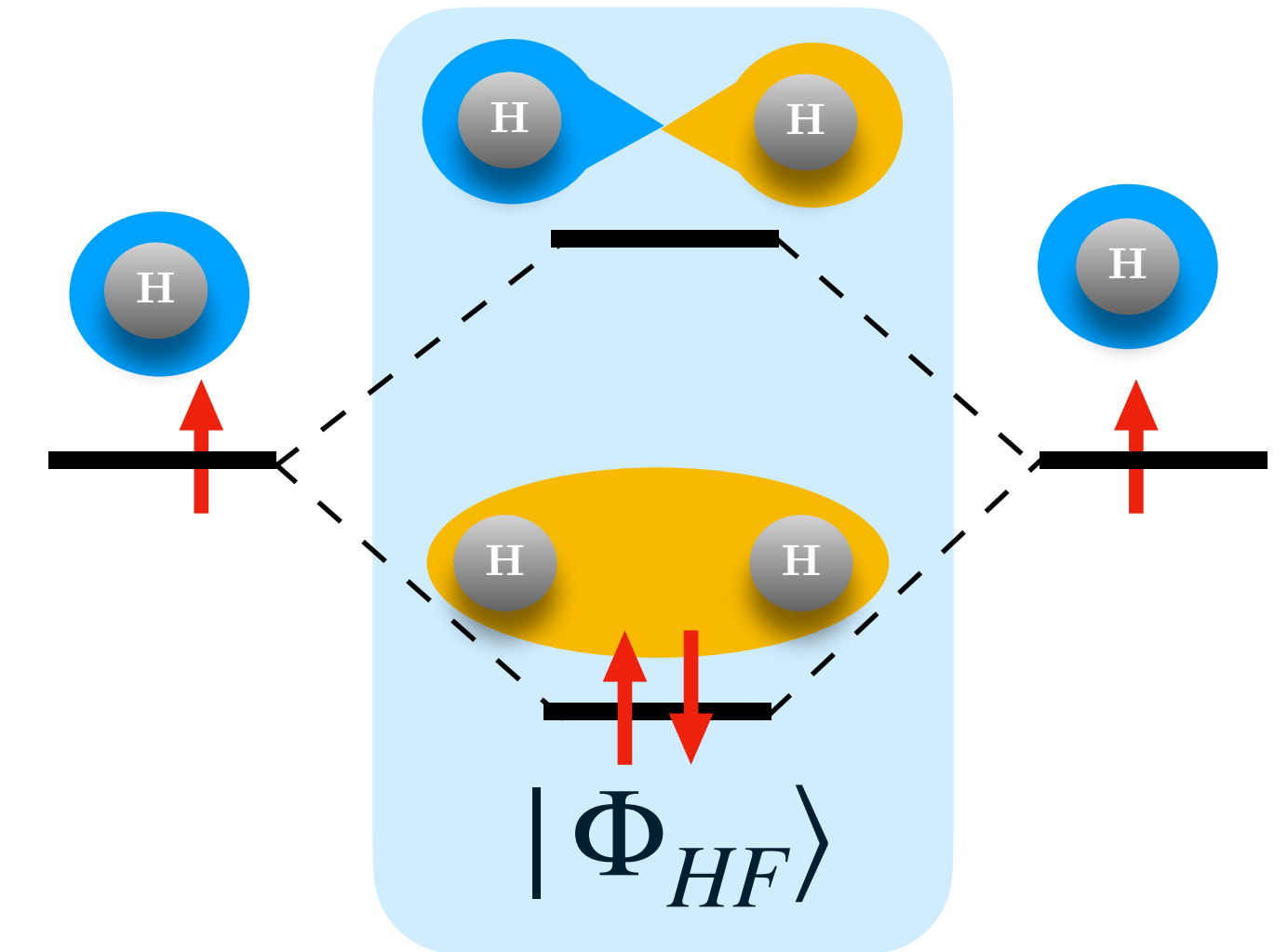
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Mean-Field Approach  
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Single Configuration  
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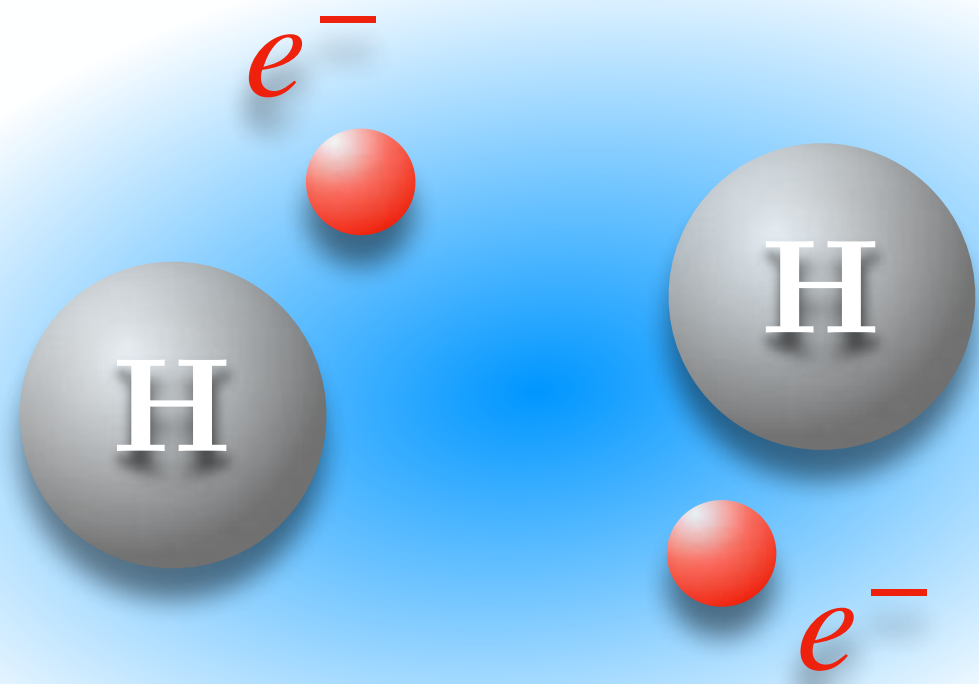
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Molecular Orbitals





## II) From quantum computing to chemistry



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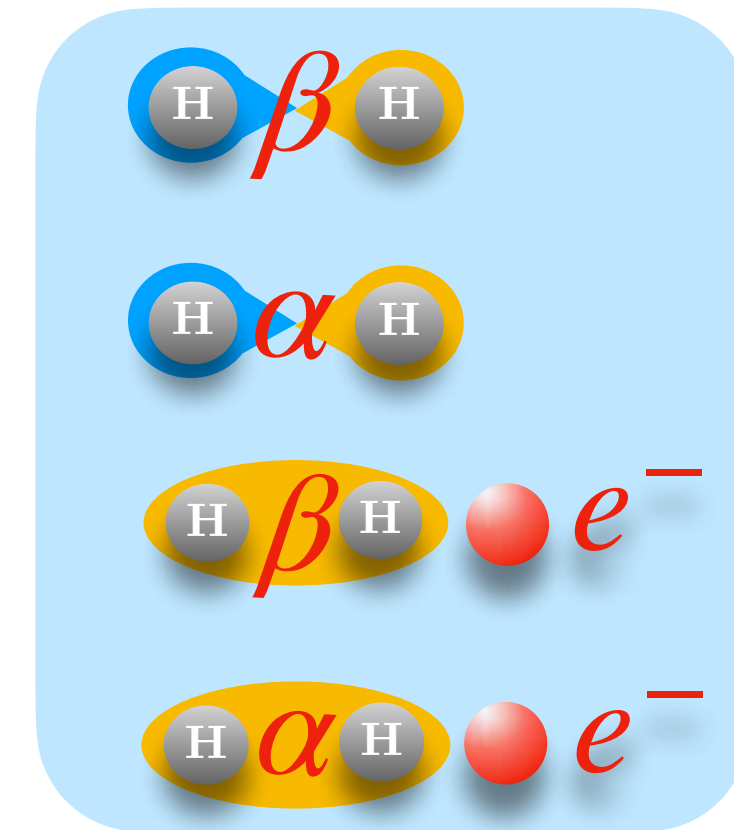
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Mean-Field Approach  
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Single Configuration  
Approximation

$$|\Psi_0\rangle \approx |\Phi_{HF}\rangle$$

Spin-Orbitals



0

0

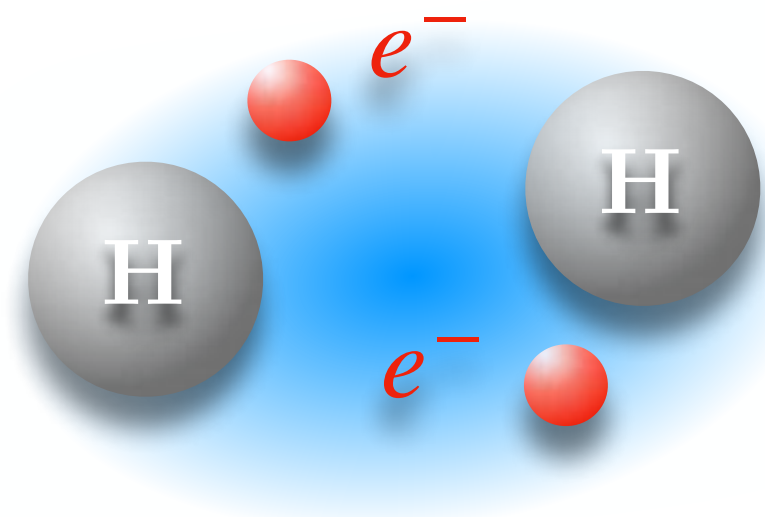
1

1

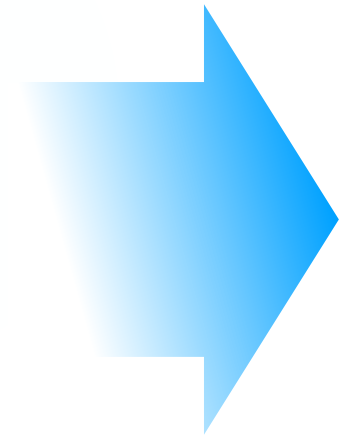
$$|\Phi_{HF}\rangle = |1100\rangle$$

## II) From quantum computing to chemistry

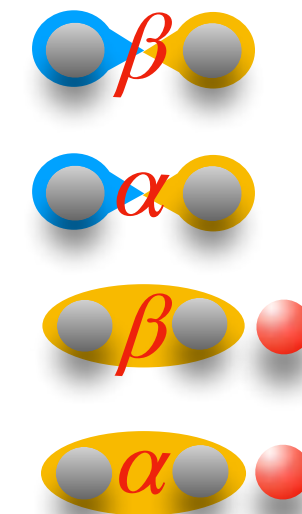
### Beyond Hartree-Fock: Full Configuration Interaction



$$H|\Psi_0\rangle = E_0|\Psi_0\rangle$$



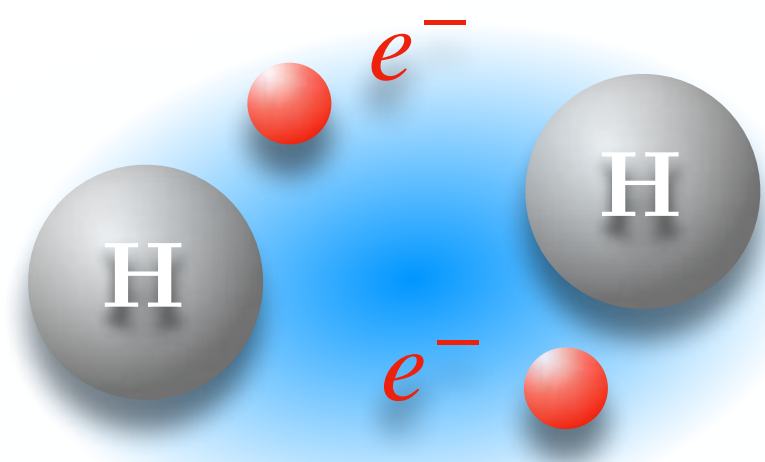
$$|\Psi_0^{FCI}\rangle =$$



$$|HF\rangle = |1100\rangle$$

## II) From quantum computing to chemistry

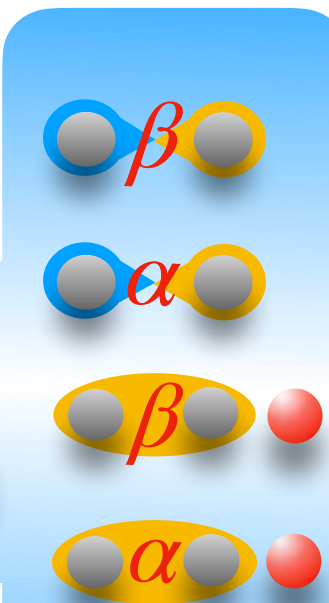
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$$H|\Psi_0\rangle = E_0|\Psi_0\rangle$$

$$|\Psi_0^{FCI}\rangle =$$

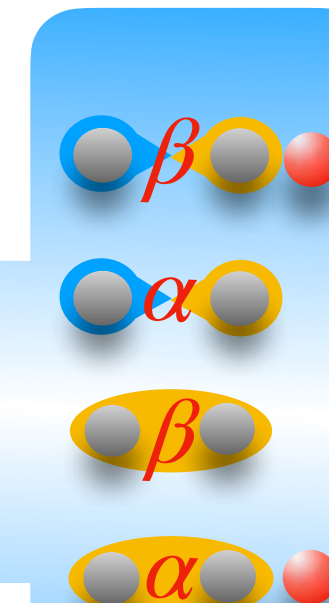
$$C_0$$



$$|HF\rangle = |1100\rangle$$

AND

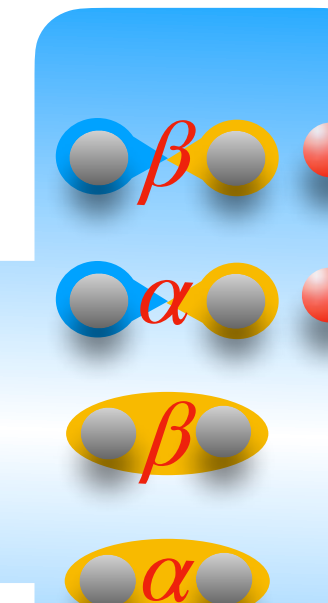
$$+ C_1$$



$$|1001\rangle$$

AND

$$+ C_2$$

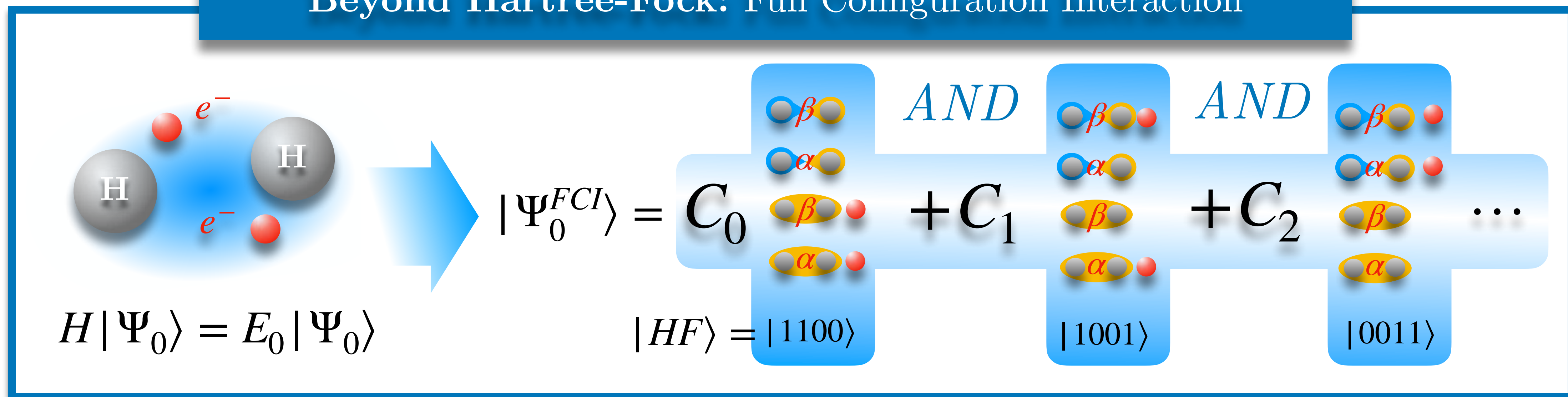


$$|0011\rangle$$

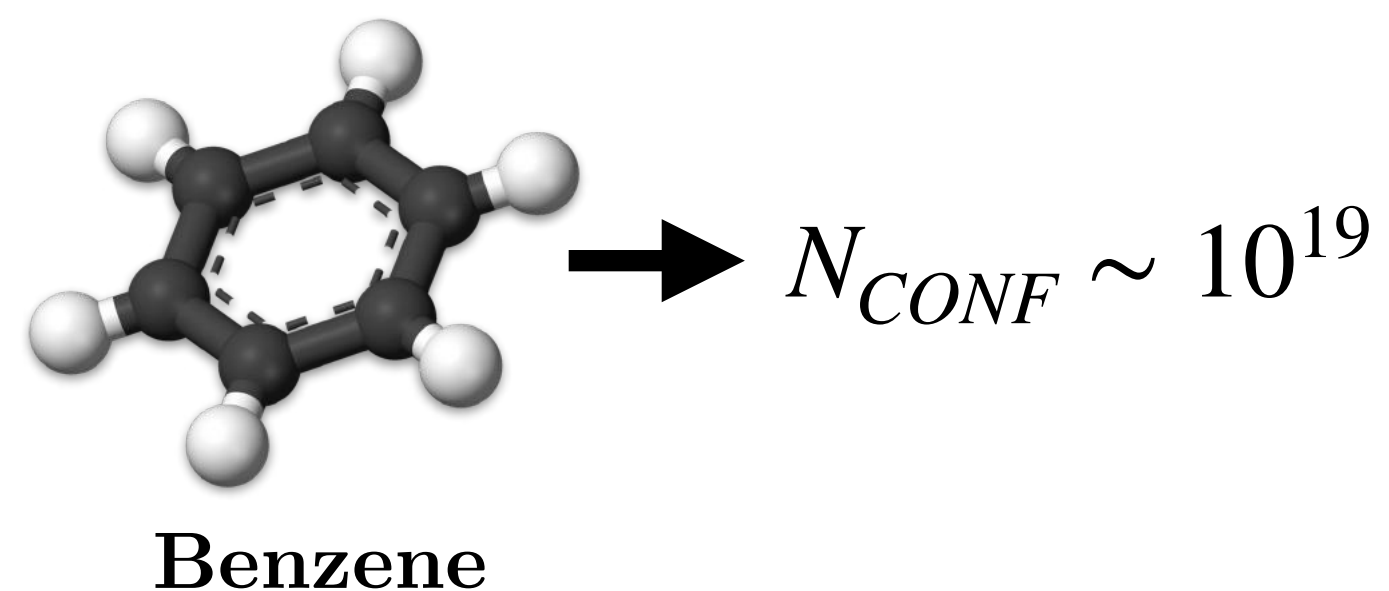
...

## II) From quantum computing to chemistry

### Beyond Hartree-Fock: Full Configuration Interaction

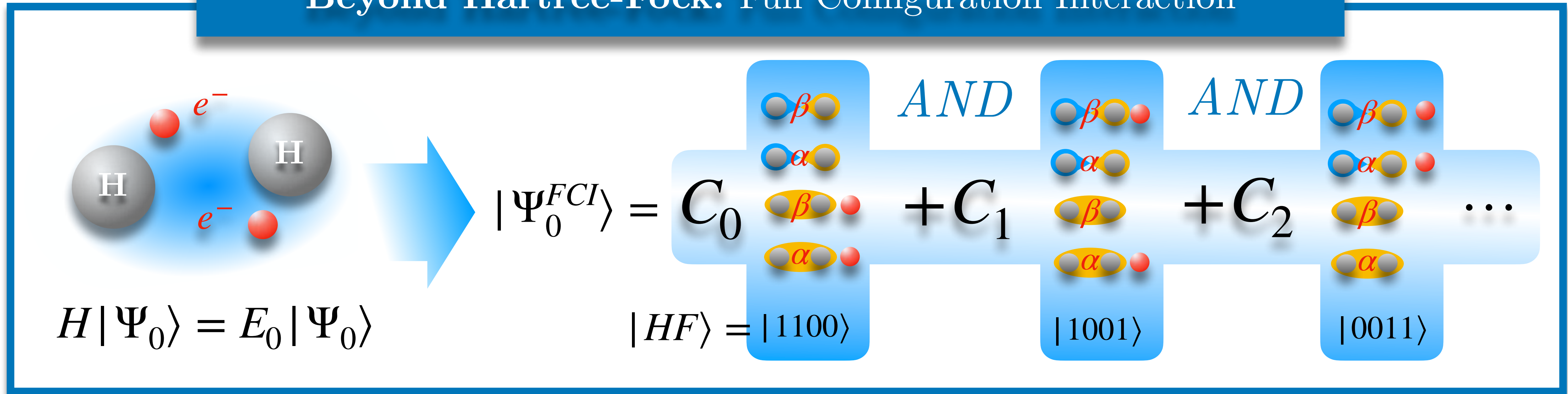


**FCI** : Ok for very small systems ...  
 But it scales **dramatically** with larger ones !

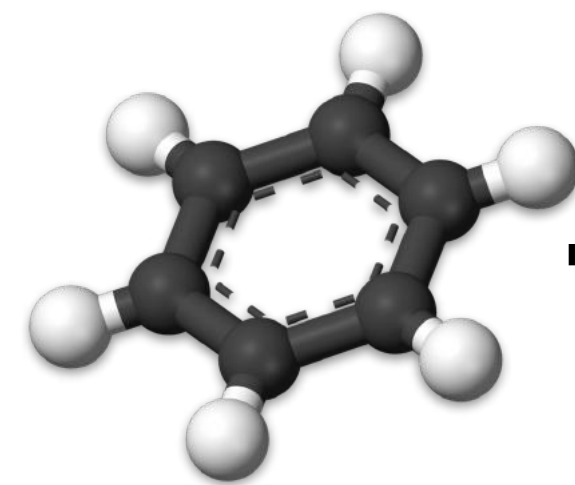


## II) From quantum computing to chemistry

### Beyond Hartree-Fock: Full Configuration Interaction



**FCI** : Ok for very small systems ...  
 But it scales **dramatically** with larger ones !



Benzene

$N_{CONF} \sim 10^{19}$

Quantum computers  
 can tackle this !

## II) From quantum computing to chemistry



*Richard P. Feynman*

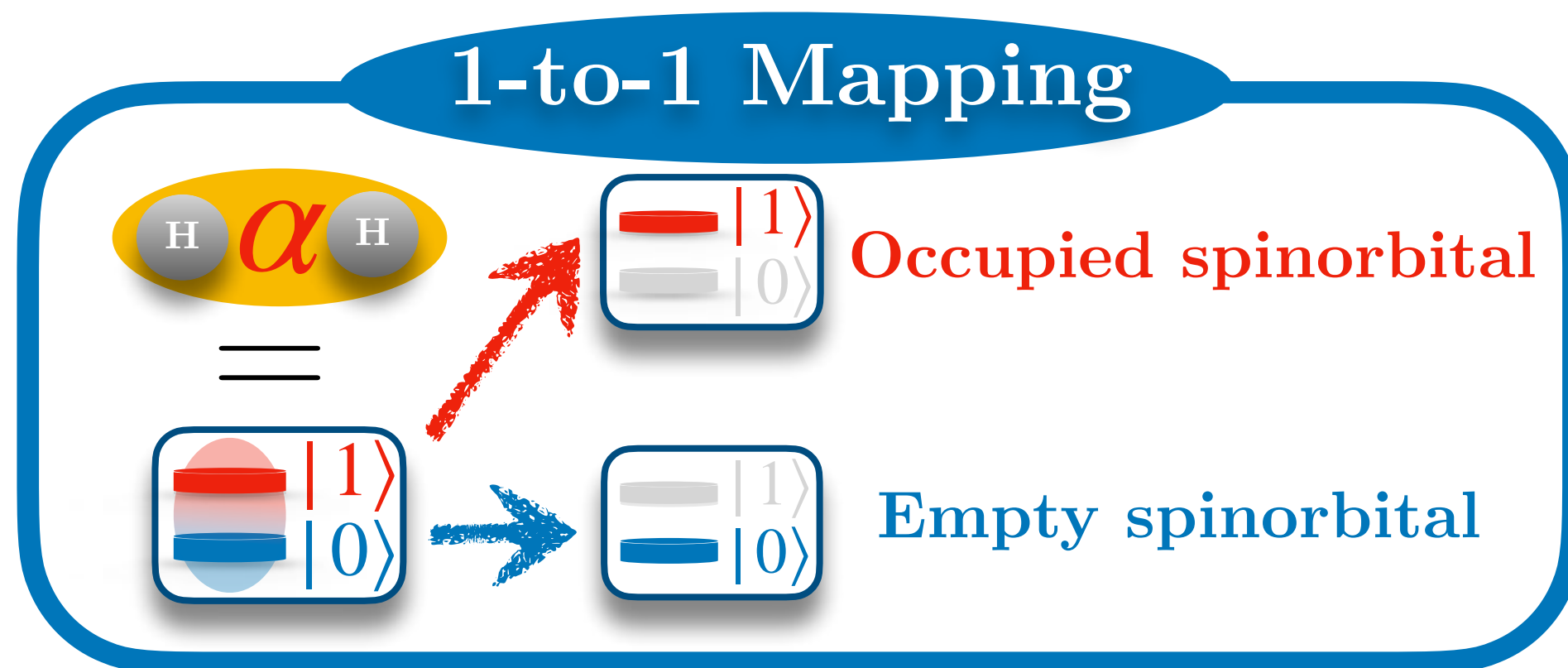
We simulate the very complex  
electronic structure problem  
(**many electrons in a molecule**)  
with a quantum computer  
containing small quantum systems  
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## II) From quantum computing to chemistry



We simulate the very complex electronic structure problem (**many electrons in a molecule**) with a quantum computer containing small quantum systems that we master (**qubits**)

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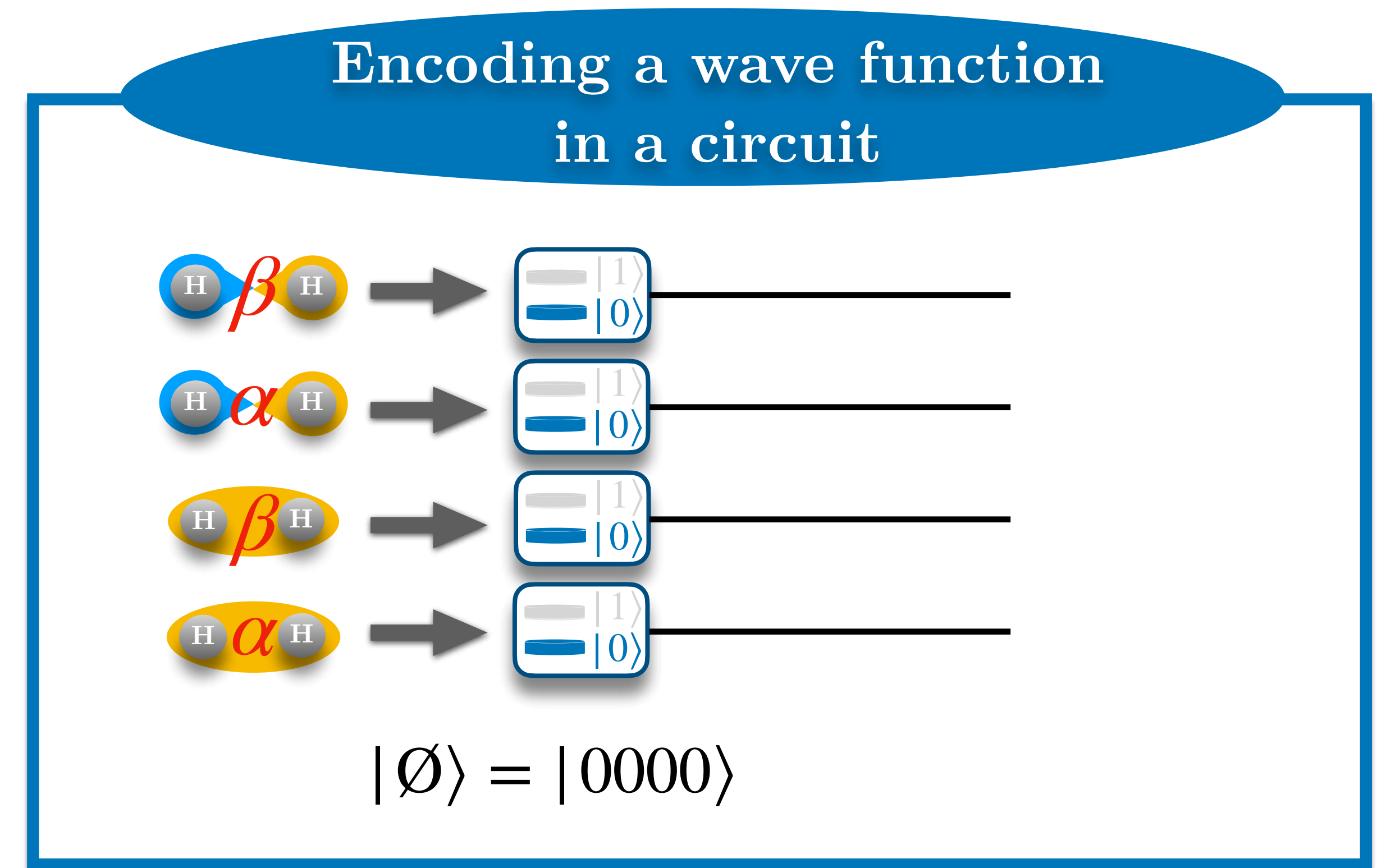
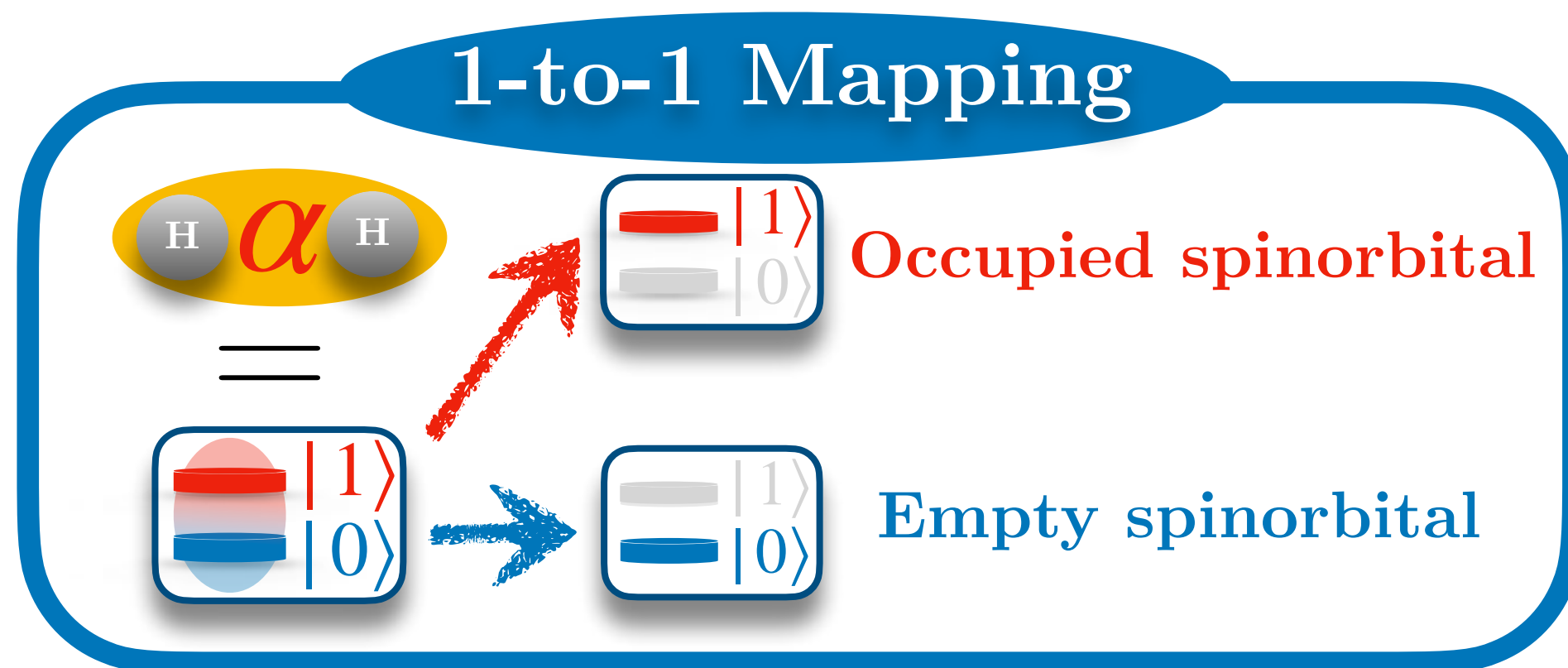


# II) From quantum computing to chemistry



We simulate the very complex electronic structure problem (many electrons in a molecule) with a quantum computer containing small quantum systems that we master (qubits)

Richard P. Feynman



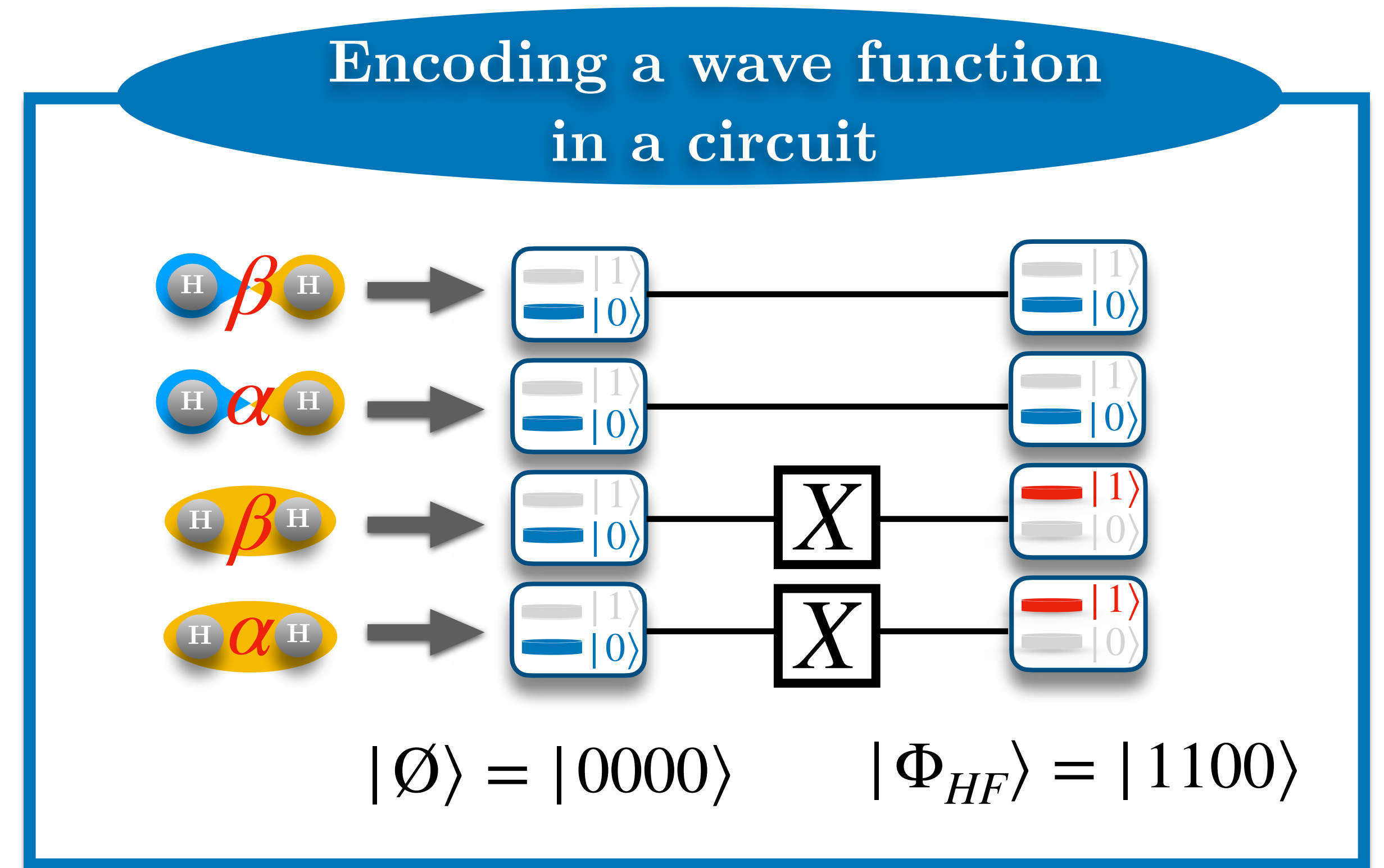
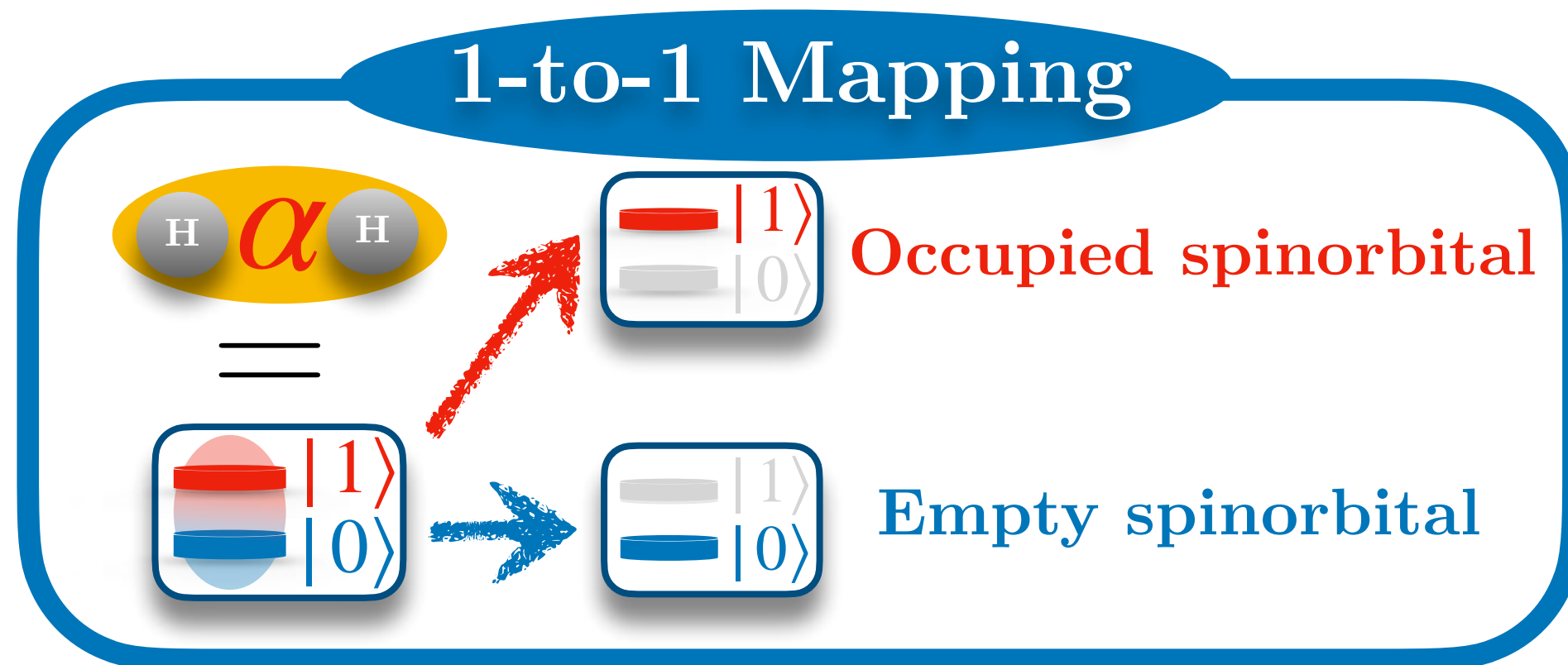


# II) From quantum computing to chemistry



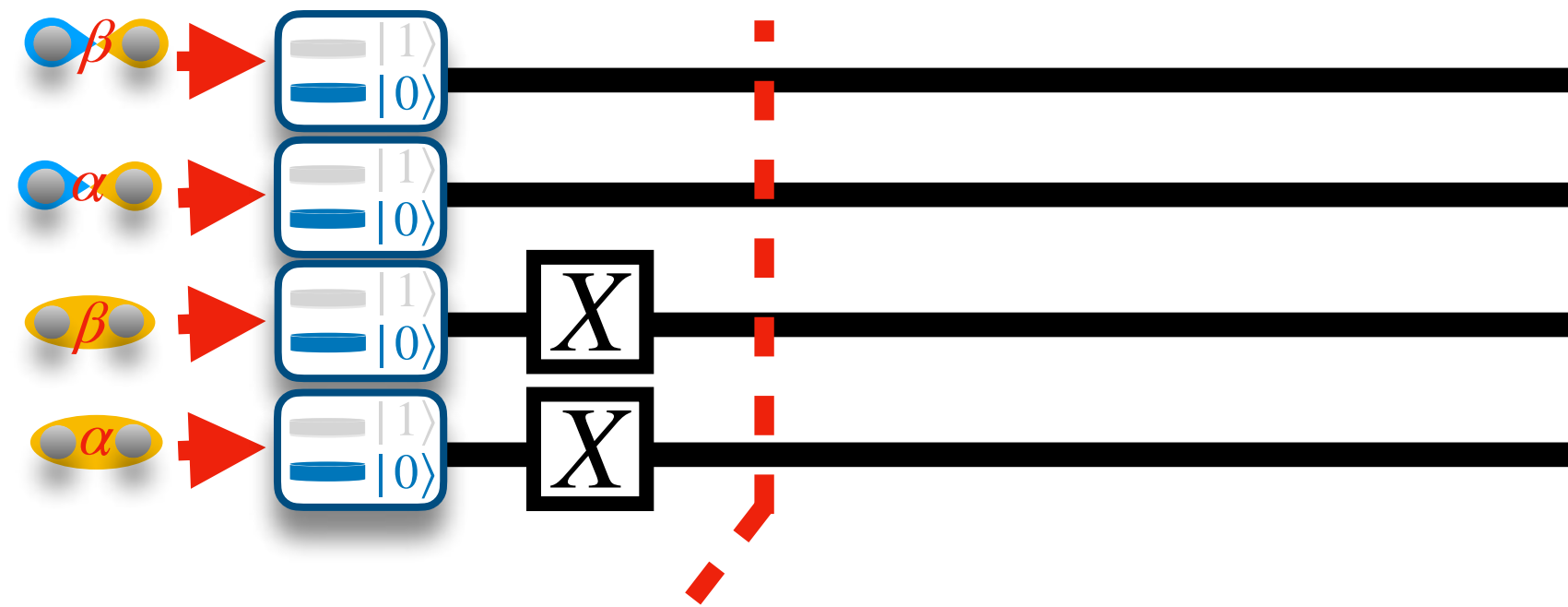
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## II) From quantum computing to chemistry

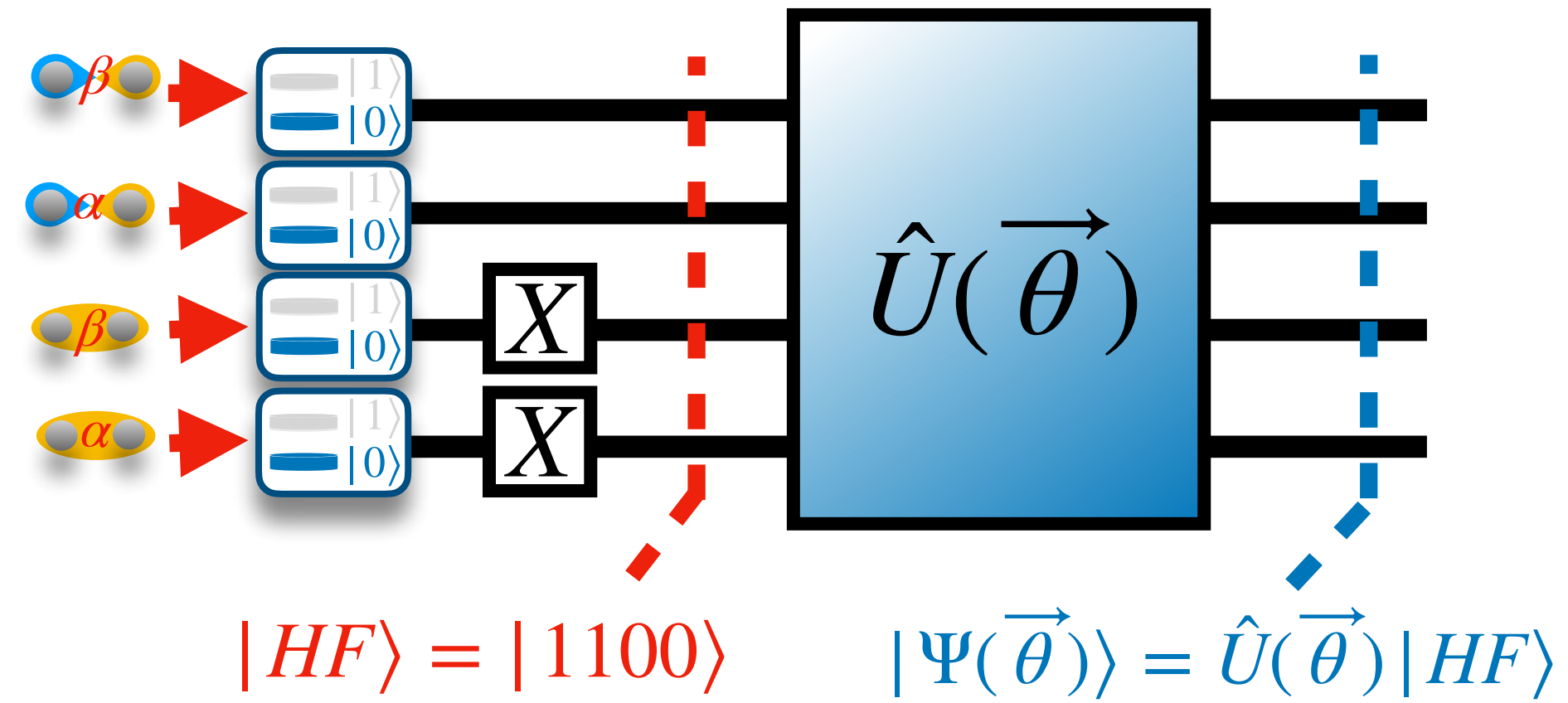
### Quantum Circuit



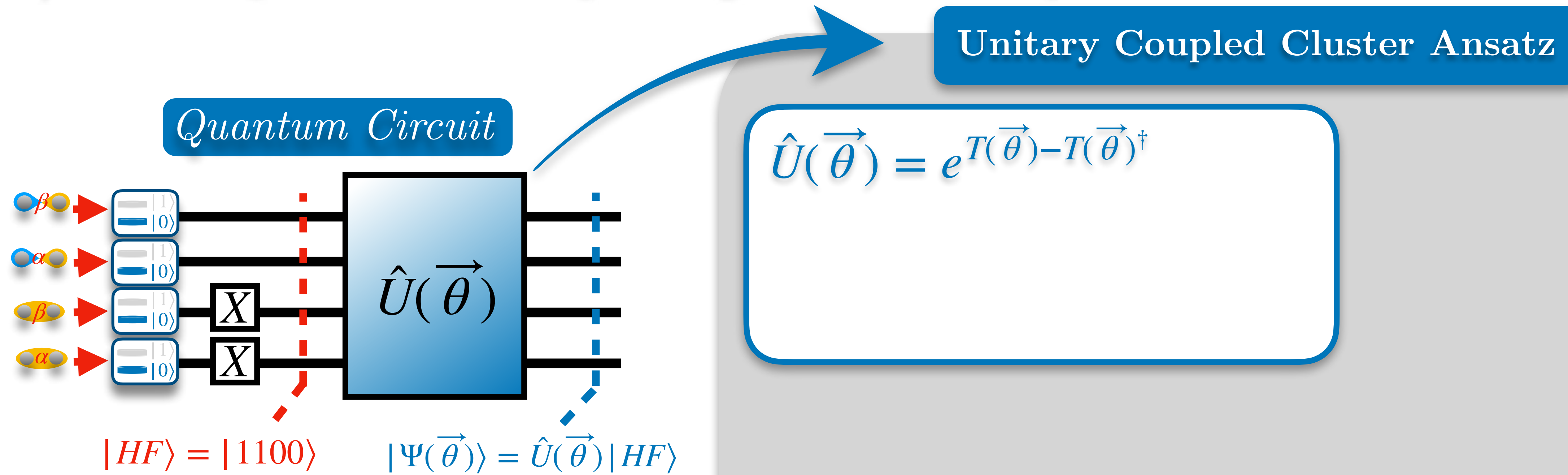
$$|HF\rangle = |1100\rangle$$

## II) From quantum computing to chemistry

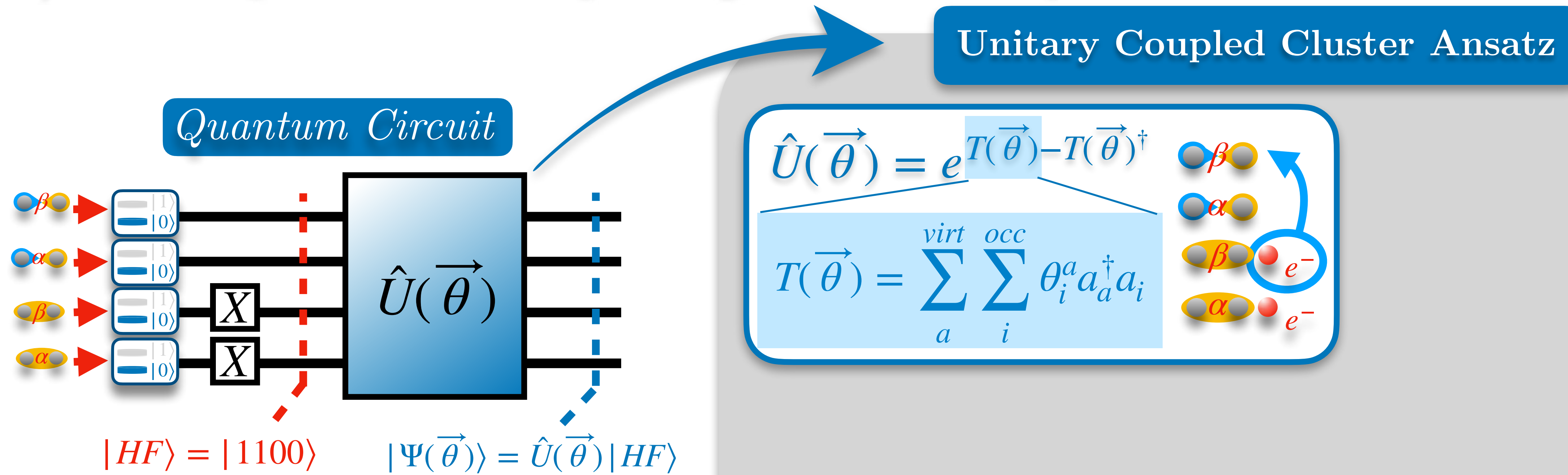
### Quantum Circuit



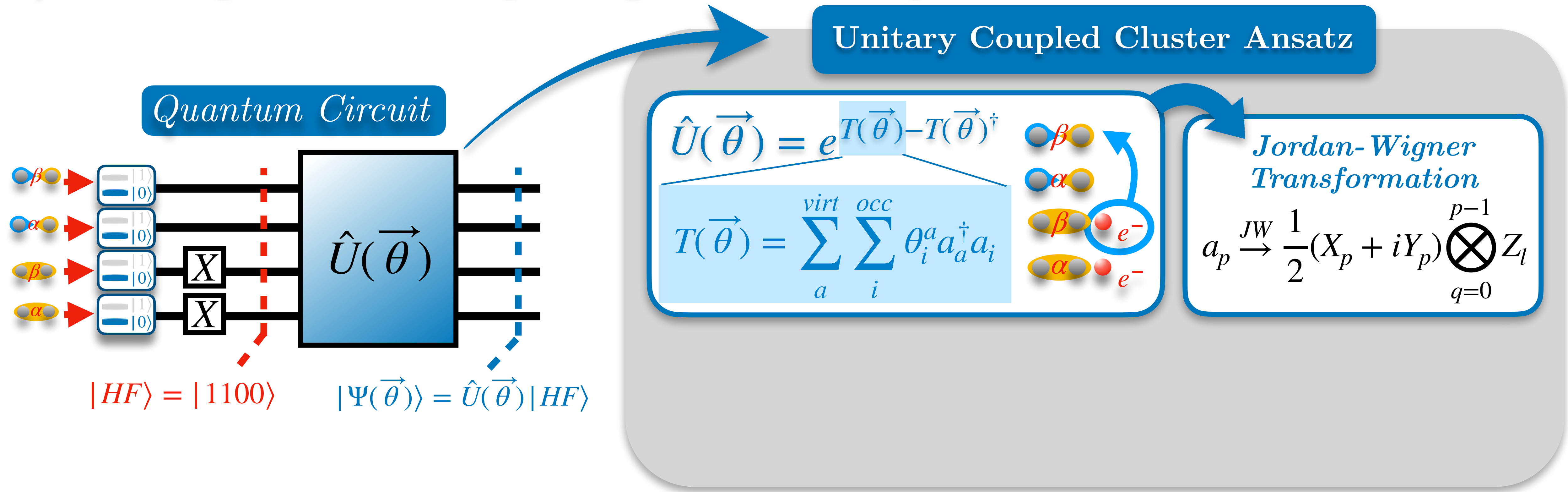
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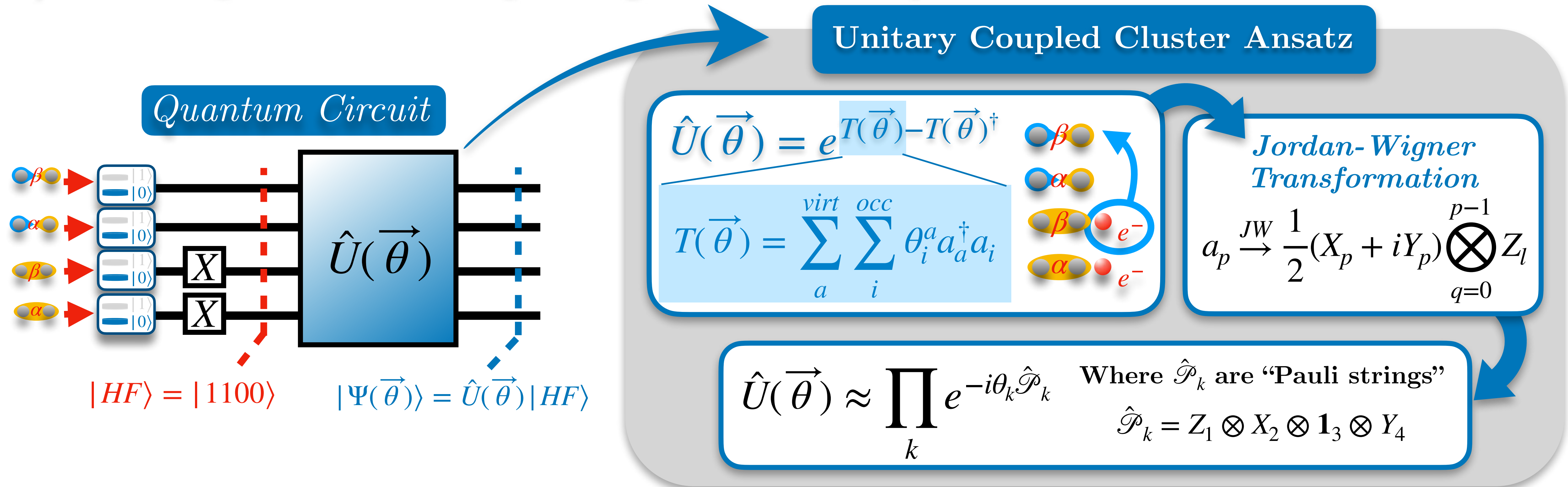
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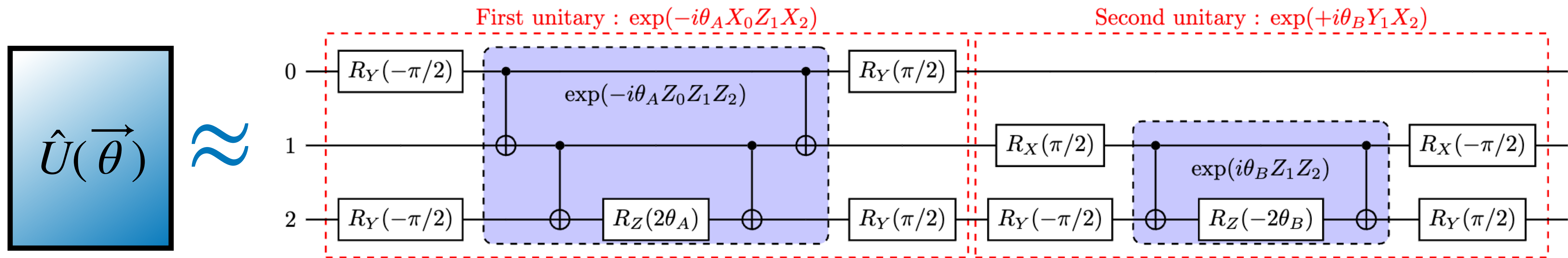
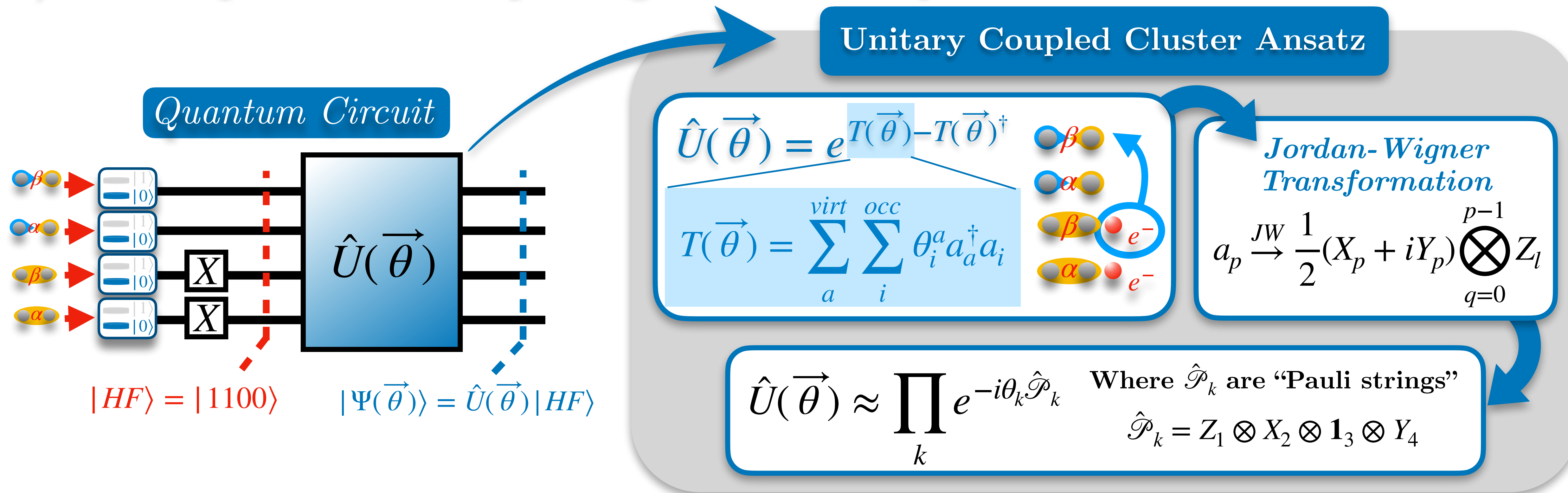
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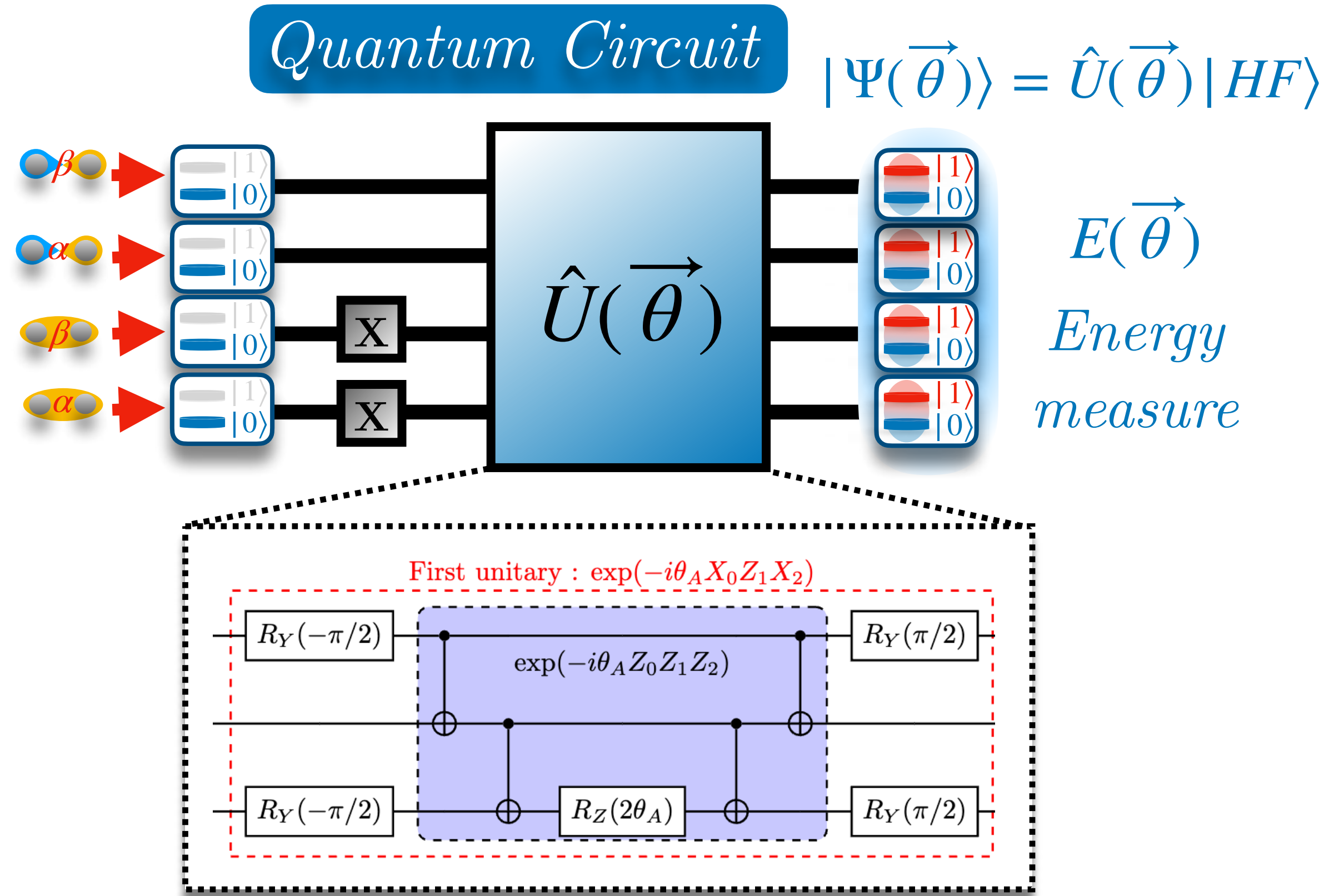
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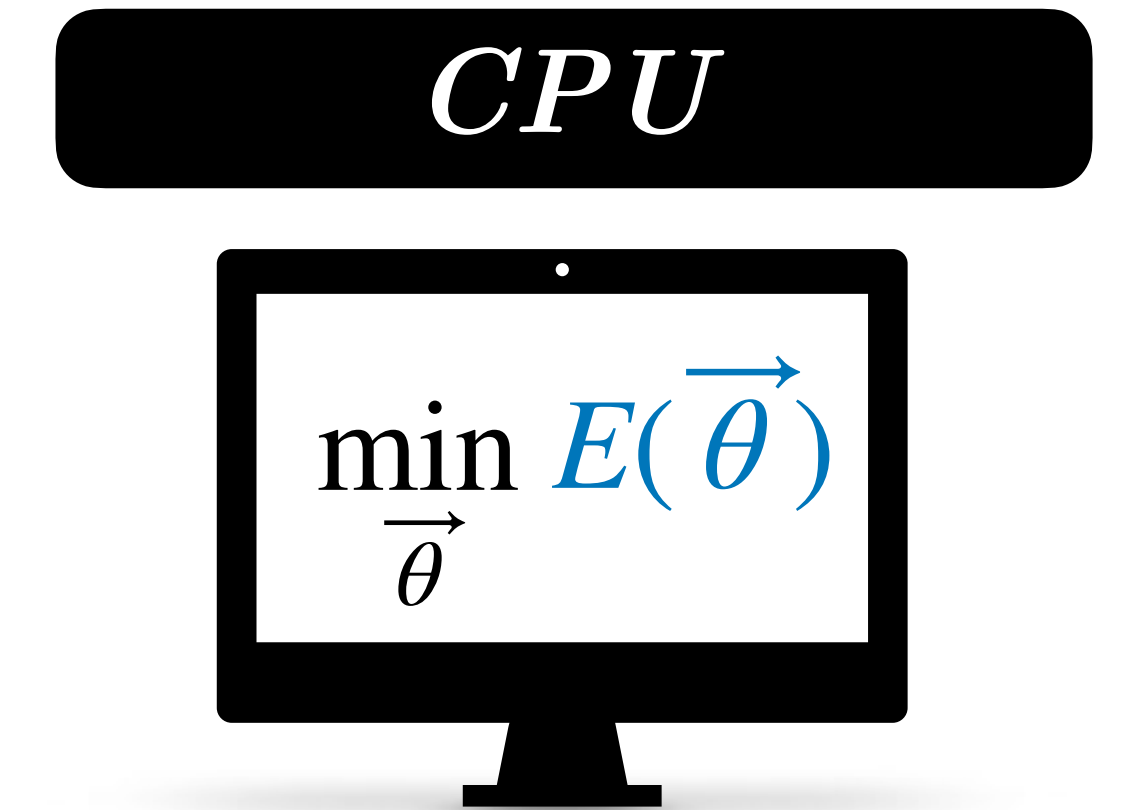
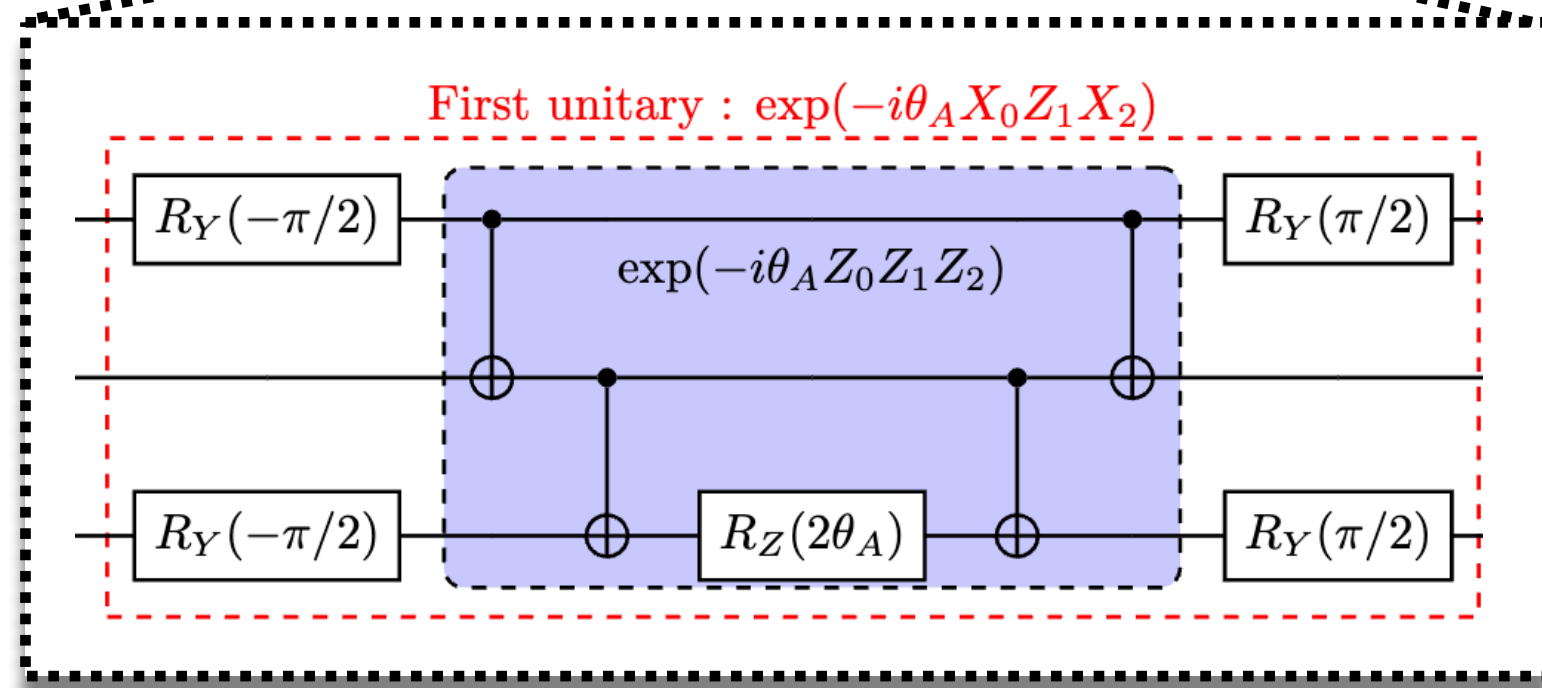
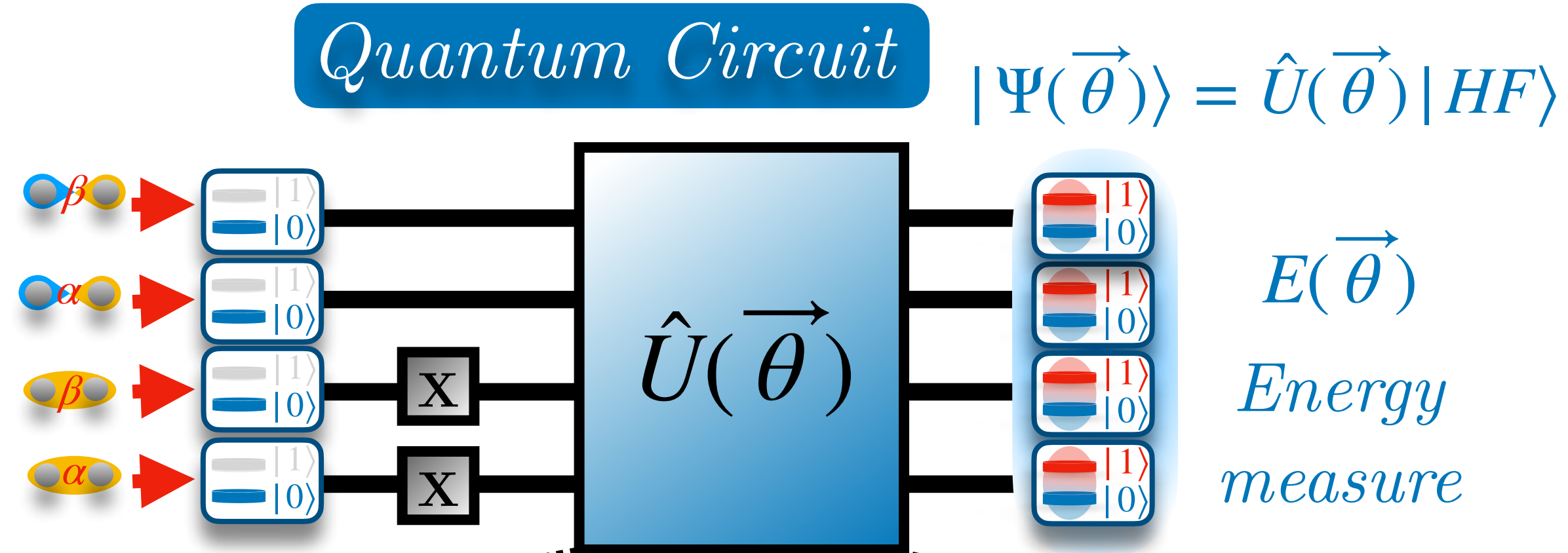
## II) From quantum computing to chemistry

*VQE : Variational Quantum Eigensolver*



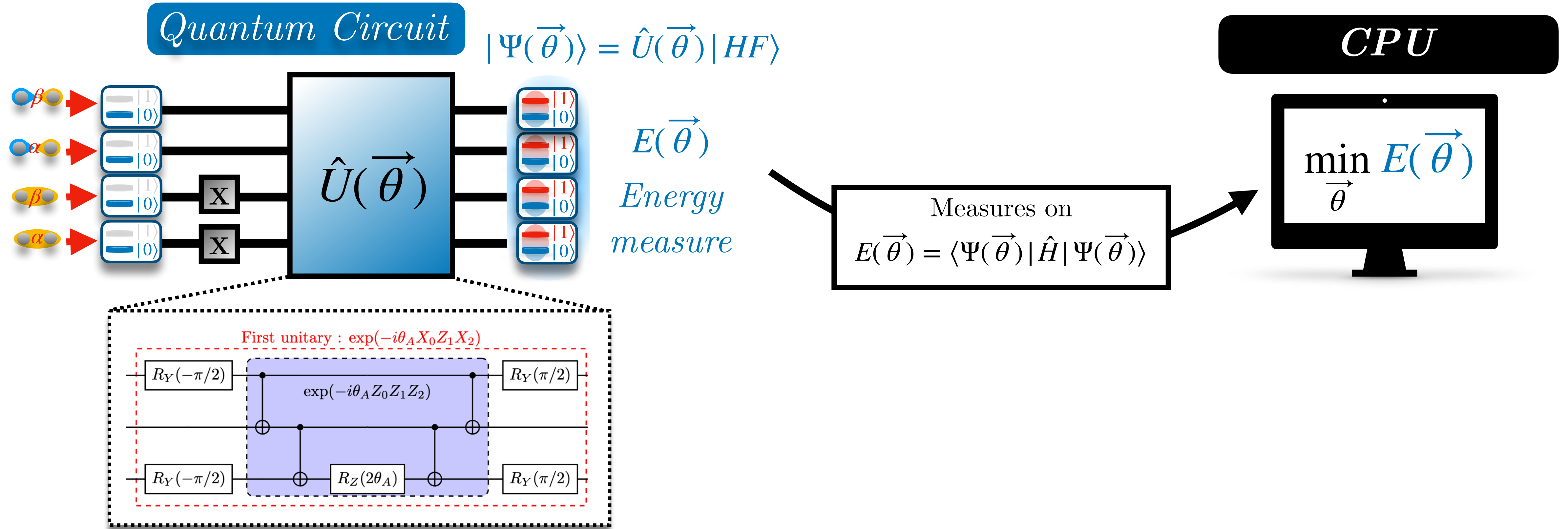
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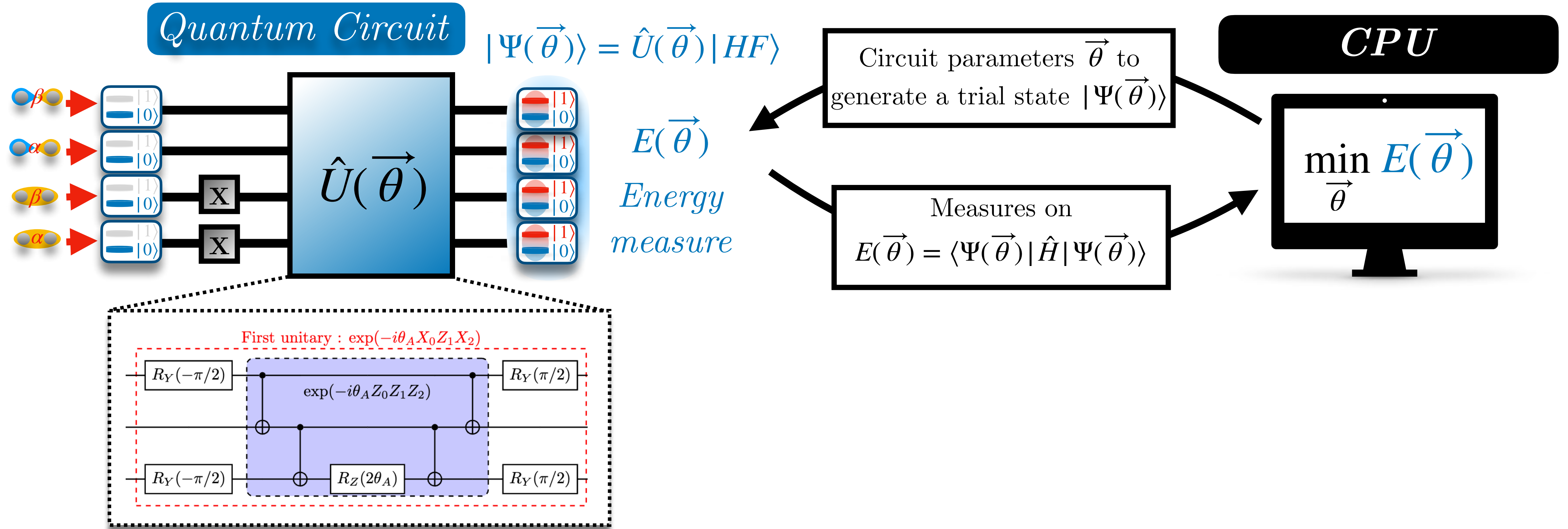
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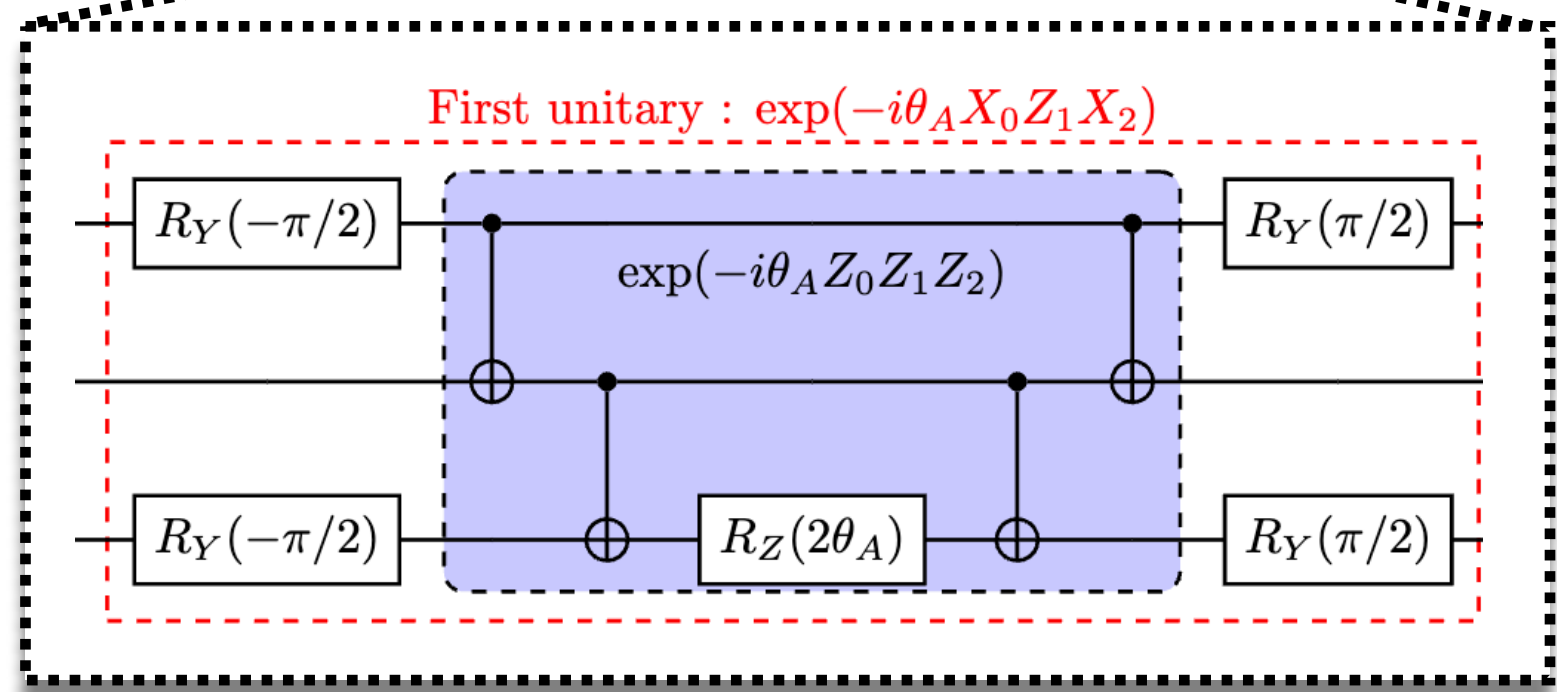
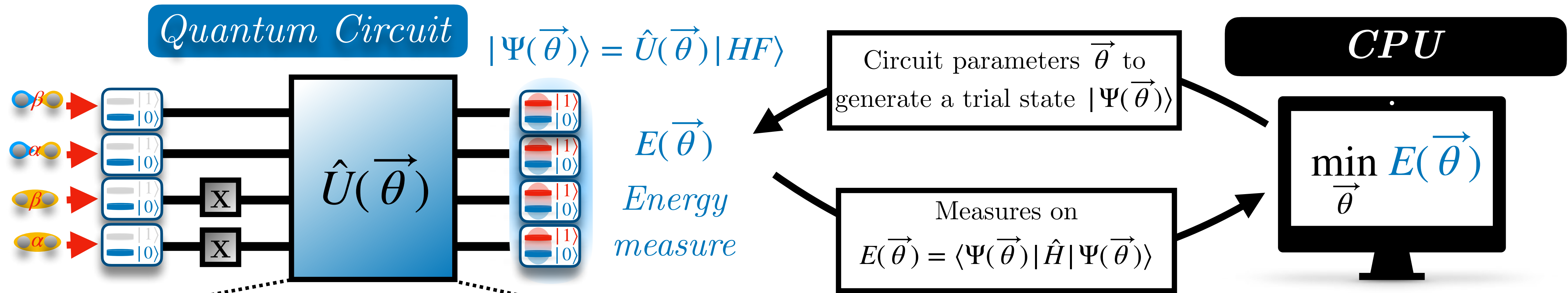
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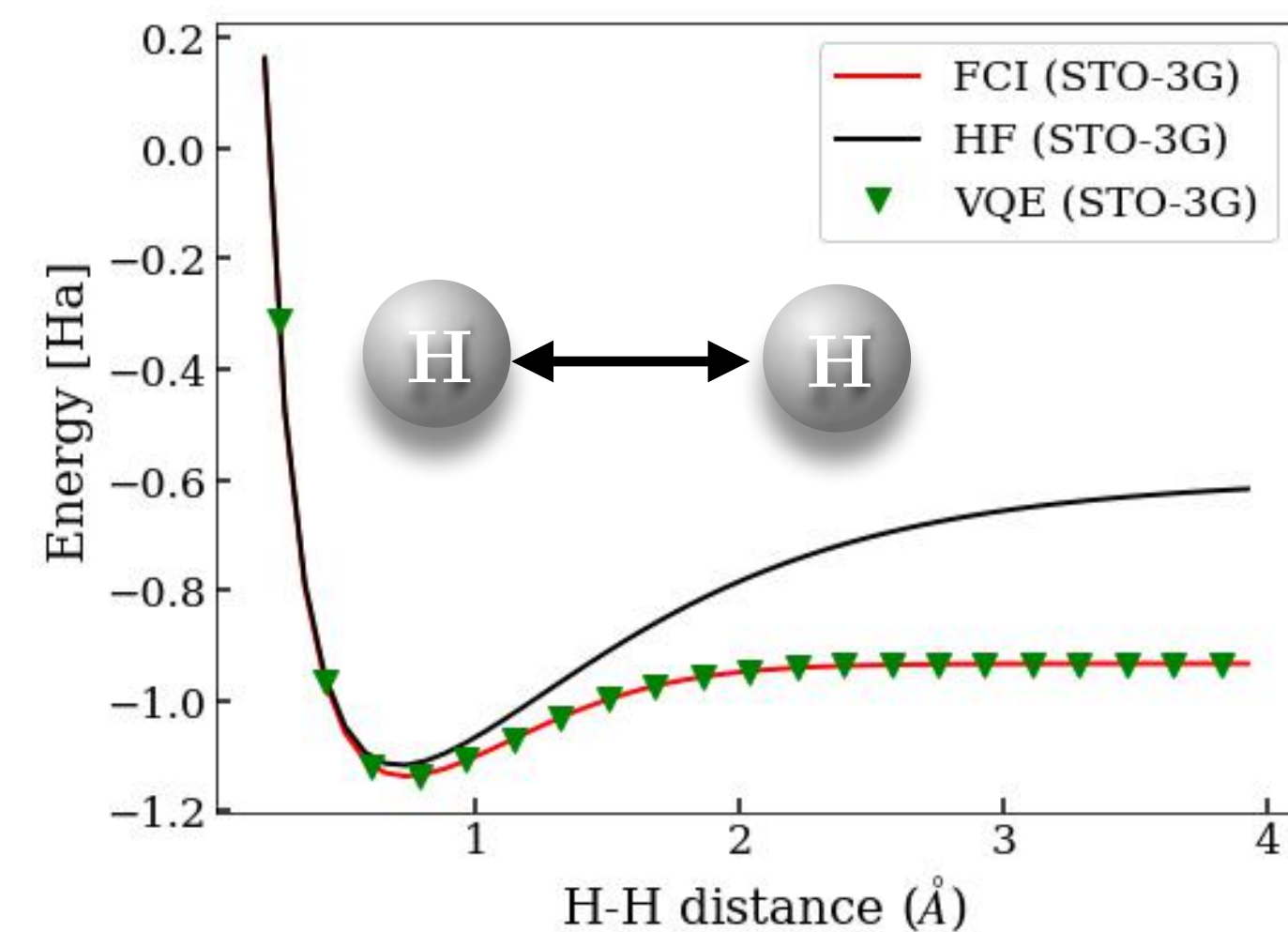


# II) From quantum computing to chemistry

## VQE : Variational Quantum Eigensolver



VQE Noiseless simulation

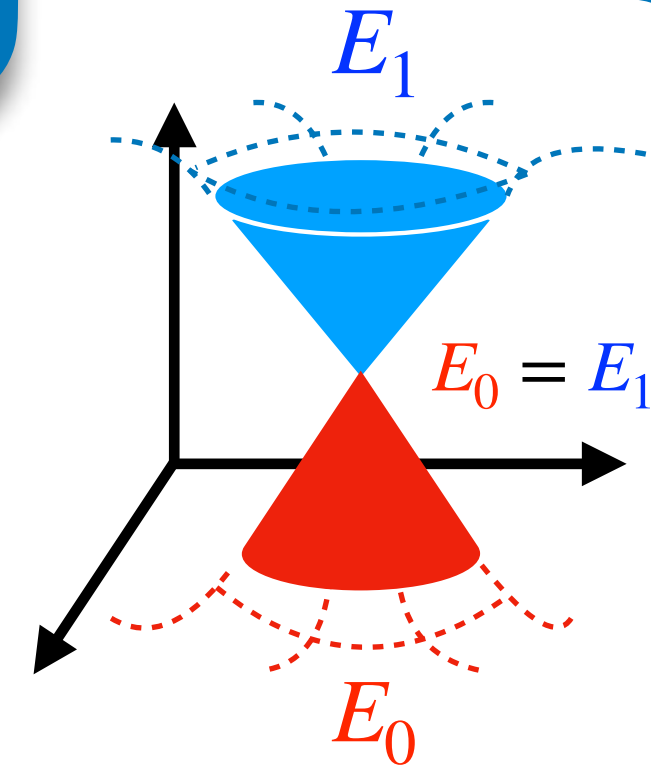


### III) Quantum algorithm for photochemistry

# III) Quantum algorithm for photochemistry

## Conical intersection

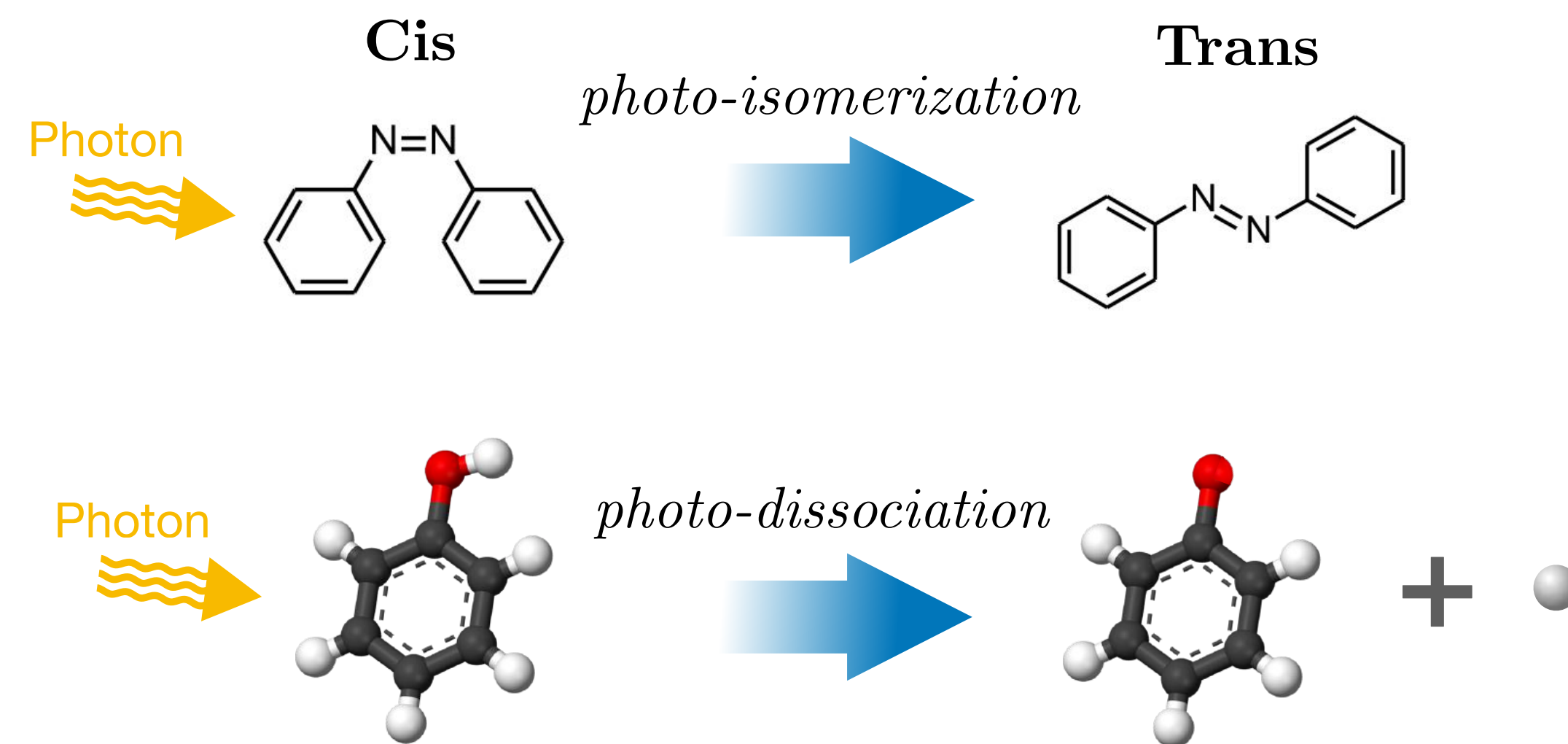
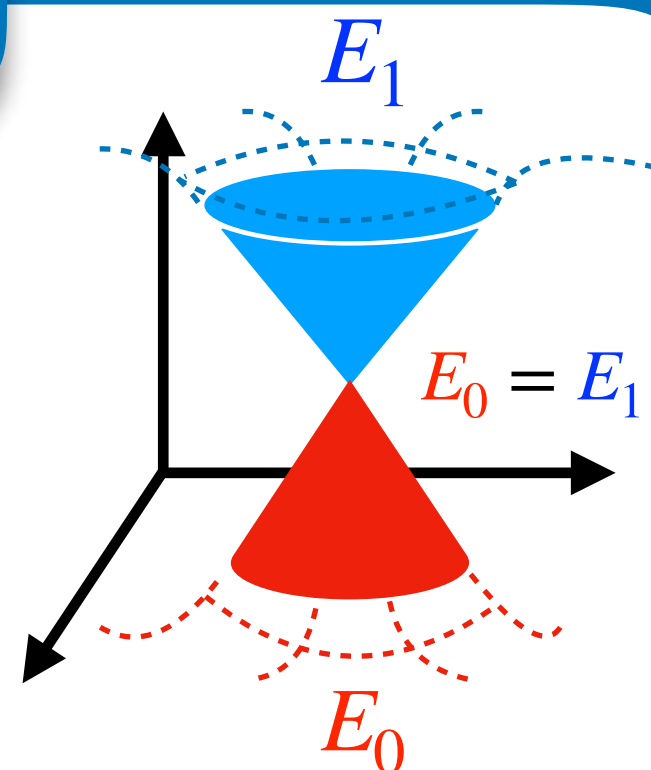
Singular point of degeneracy  
connecting two  
Potential Energy Surfaces



# III) Quantum algorithm for photochemistry

## Conical intersection

Singular point of degeneracy  
connecting two  
Potential Energy Surfaces

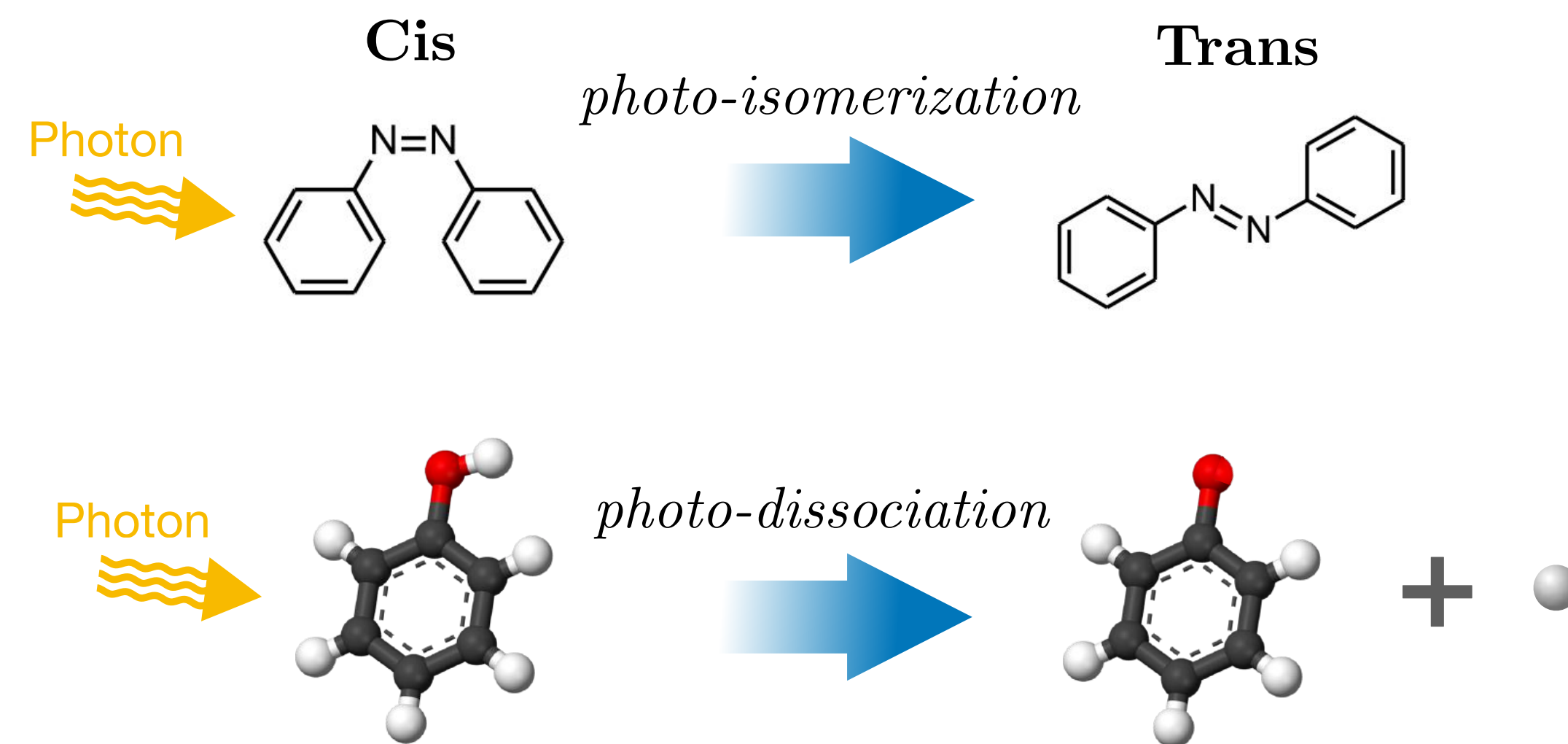
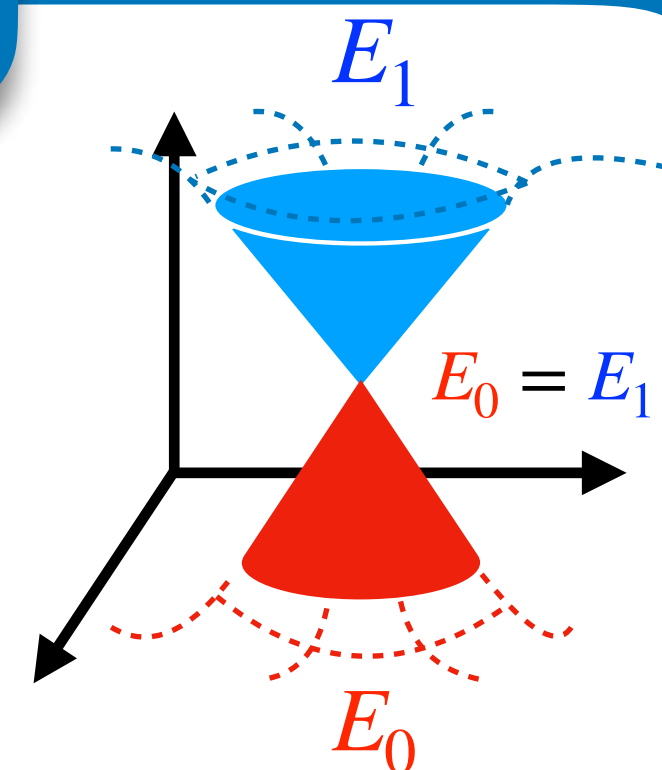




# III) Quantum algorithm for photochemistry

## Conical intersection

Singular point of degeneracy connecting two Potential Energy Surfaces



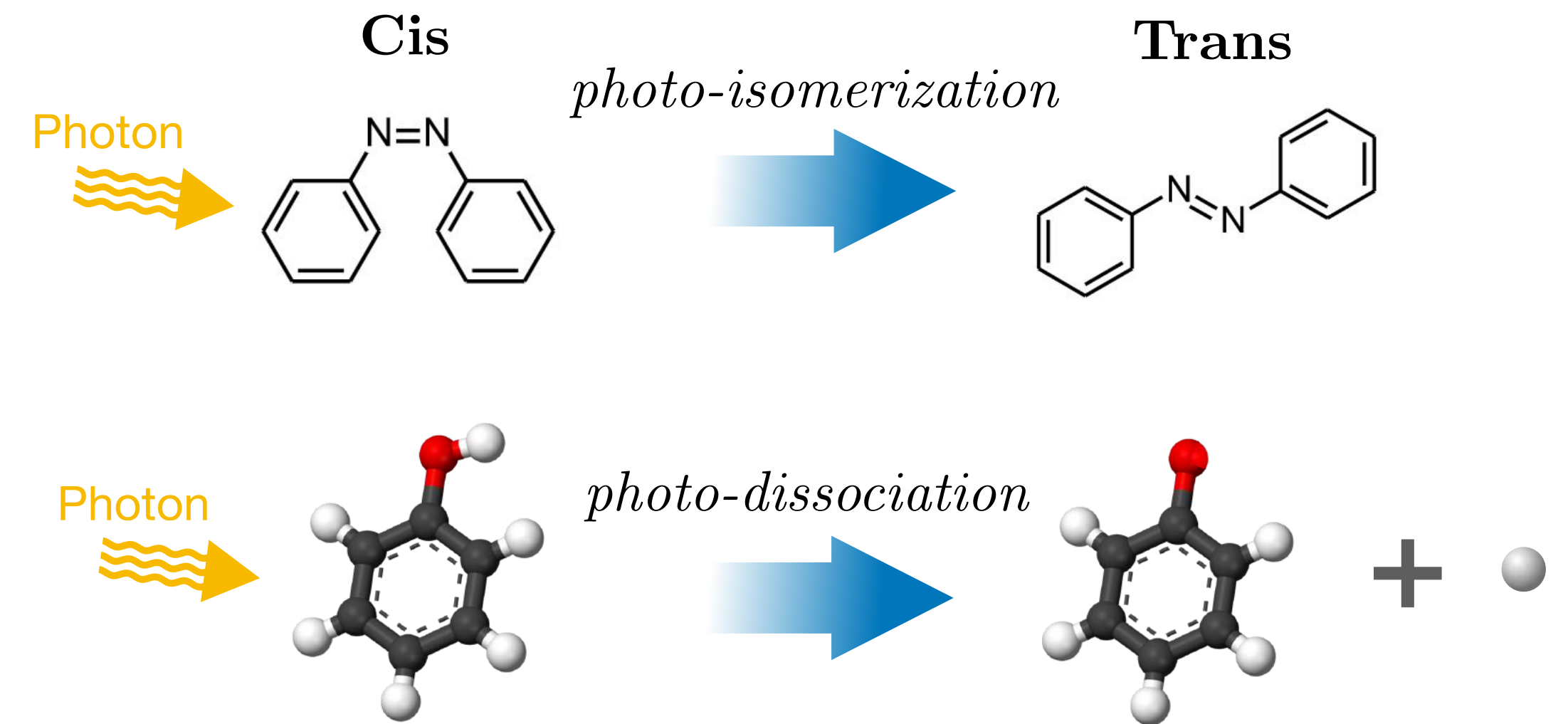
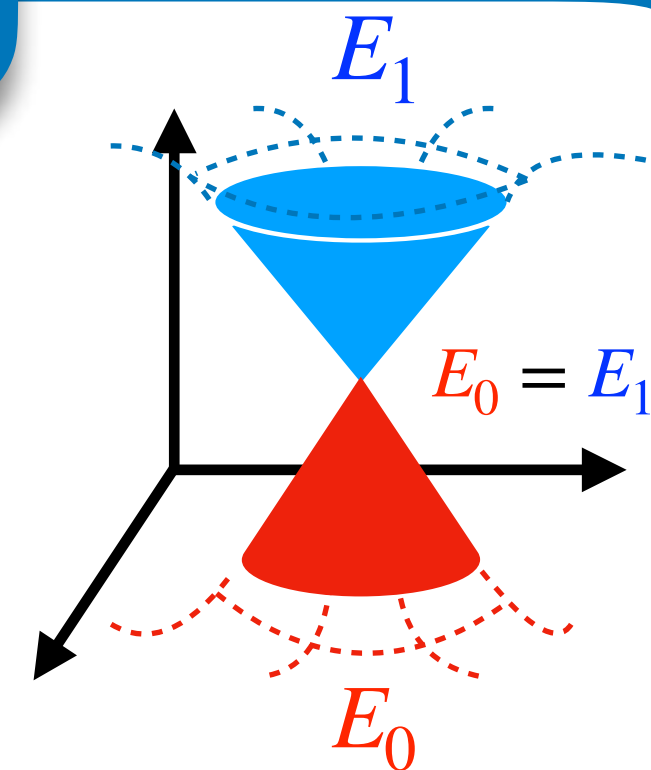
Challenge !

Democratic treatment of Ground + Excited states

# III) Quantum algorithm for photochemistry

## Conical intersection

Singular point of degeneracy connecting two Potential Energy Surfaces



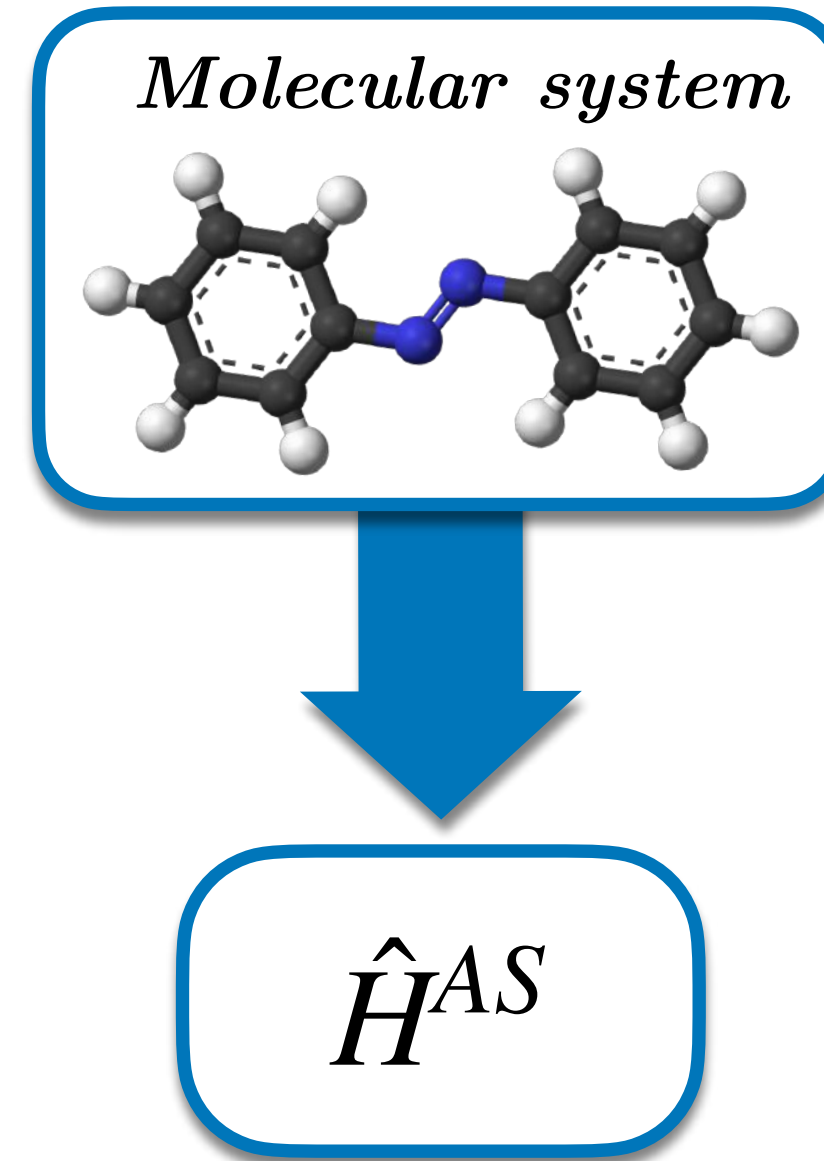
Challenge !

Democratic treatment of Ground + Excited states

## SA-OO-VQE: State-Averaged Orbital-Optimized VQE

- Treats on an equal footing ensemble of states
- Provides useful data for photochemistry studies (*e.g.* PES, gradients and NAC)

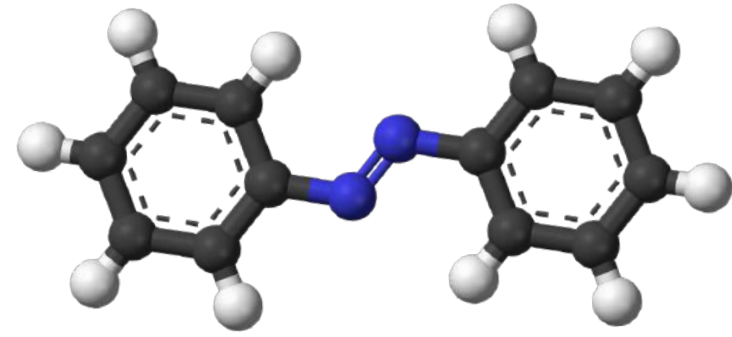
# III) Quantum algorithm for photochemistry



# III) Quantum algorithm for photochemistry

State-Averaged VQE

*Molecular system*

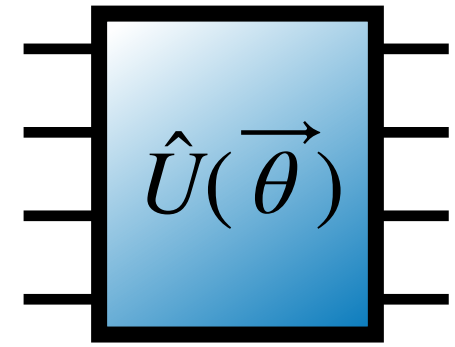


$$\hat{H}^{AS}$$

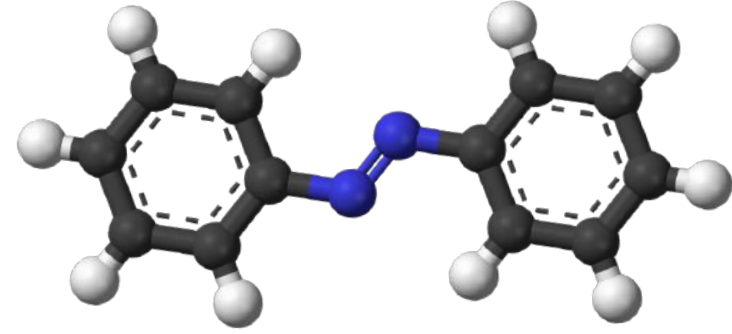


# III) Quantum algorithm for photochemistry

State-Averaged VQE


$$\hat{U}(\vec{\theta})$$

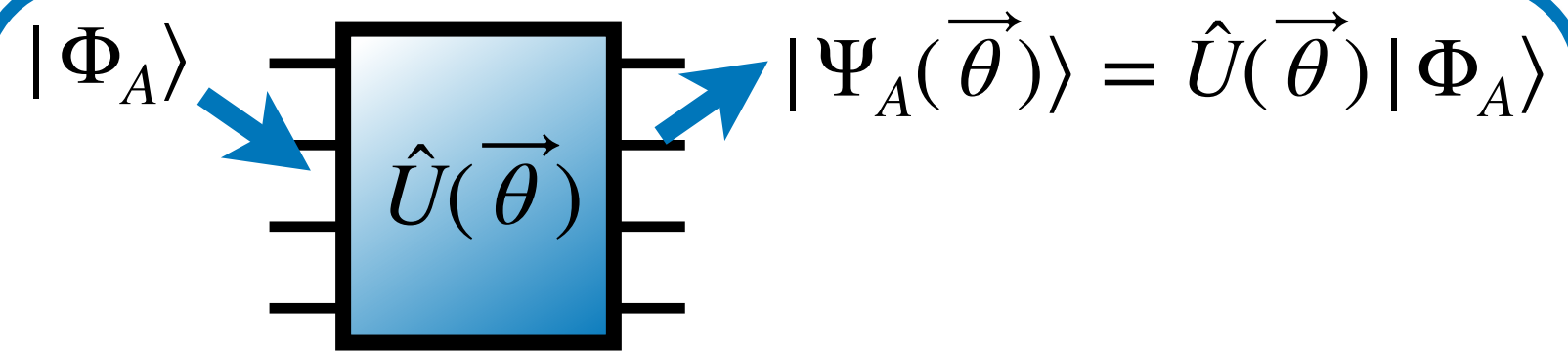
*Molecular system*



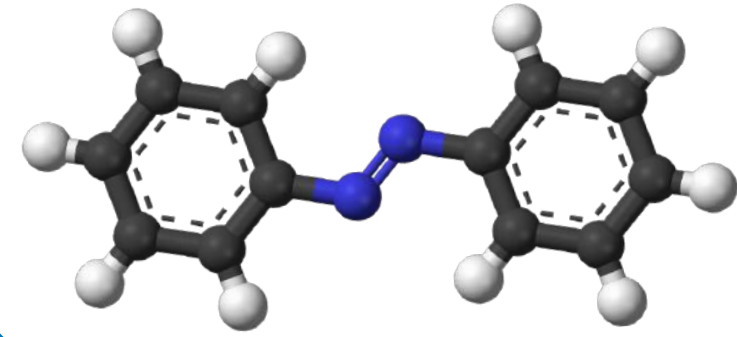
$$\hat{H}^{AS}$$

# III) Quantum algorithm for photochemistry

## State-Averaged VQE



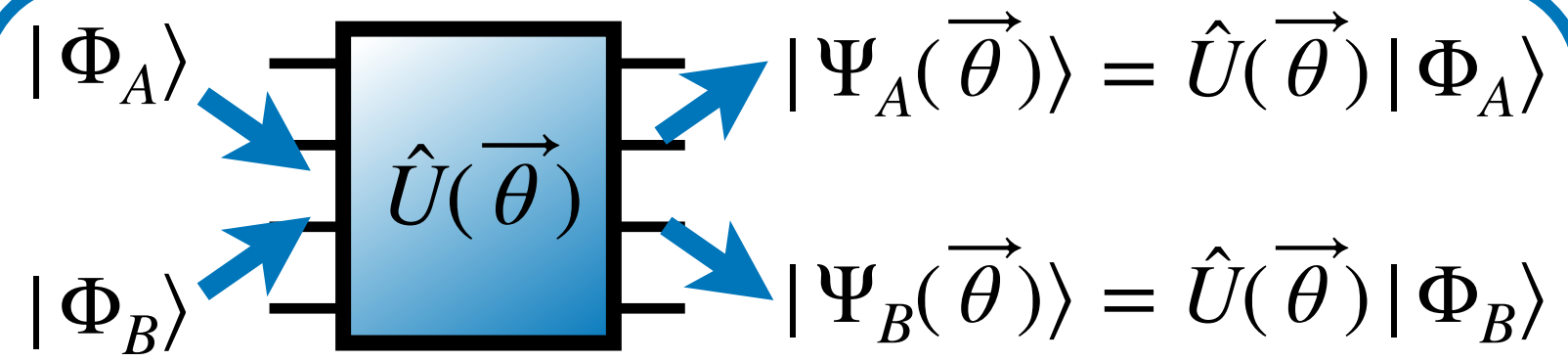
*Molecular system*



$\hat{H}^{AS}$

# III) Quantum algorithm for photochemistry

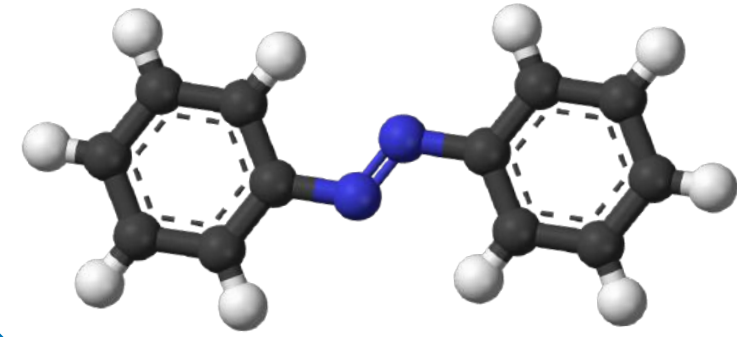
## State-Averaged VQE



The diagram illustrates the State-Averaged VQE process. It features a central box labeled  $\hat{U}(\vec{\theta})$  with two input arrows on the left and two output arrows on the right. The inputs are labeled  $|\Phi_A\rangle$  and  $|\Phi_B\rangle$ . The outputs are labeled  $|\Psi_A(\vec{\theta})\rangle = \hat{U}(\vec{\theta})|\Phi_A\rangle$  and  $|\Psi_B(\vec{\theta})\rangle = \hat{U}(\vec{\theta})|\Phi_B\rangle$ . A large blue arrow points from the right towards this box, originating from a box labeled  $\hat{H}^{AS}$ . Above this, a box labeled "Molecular system" contains a ball-and-stick model of a molecule with two benzene rings connected by a nitrogen bridge. A blue arrow points from this box down to the  $\hat{H}^{AS}$  box.

$$|\Phi_A\rangle \rightarrow \hat{U}(\vec{\theta}) \rightarrow |\Psi_A(\vec{\theta})\rangle = \hat{U}(\vec{\theta})|\Phi_A\rangle$$
$$|\Phi_B\rangle \rightarrow \hat{U}(\vec{\theta}) \rightarrow |\Psi_B(\vec{\theta})\rangle = \hat{U}(\vec{\theta})|\Phi_B\rangle$$

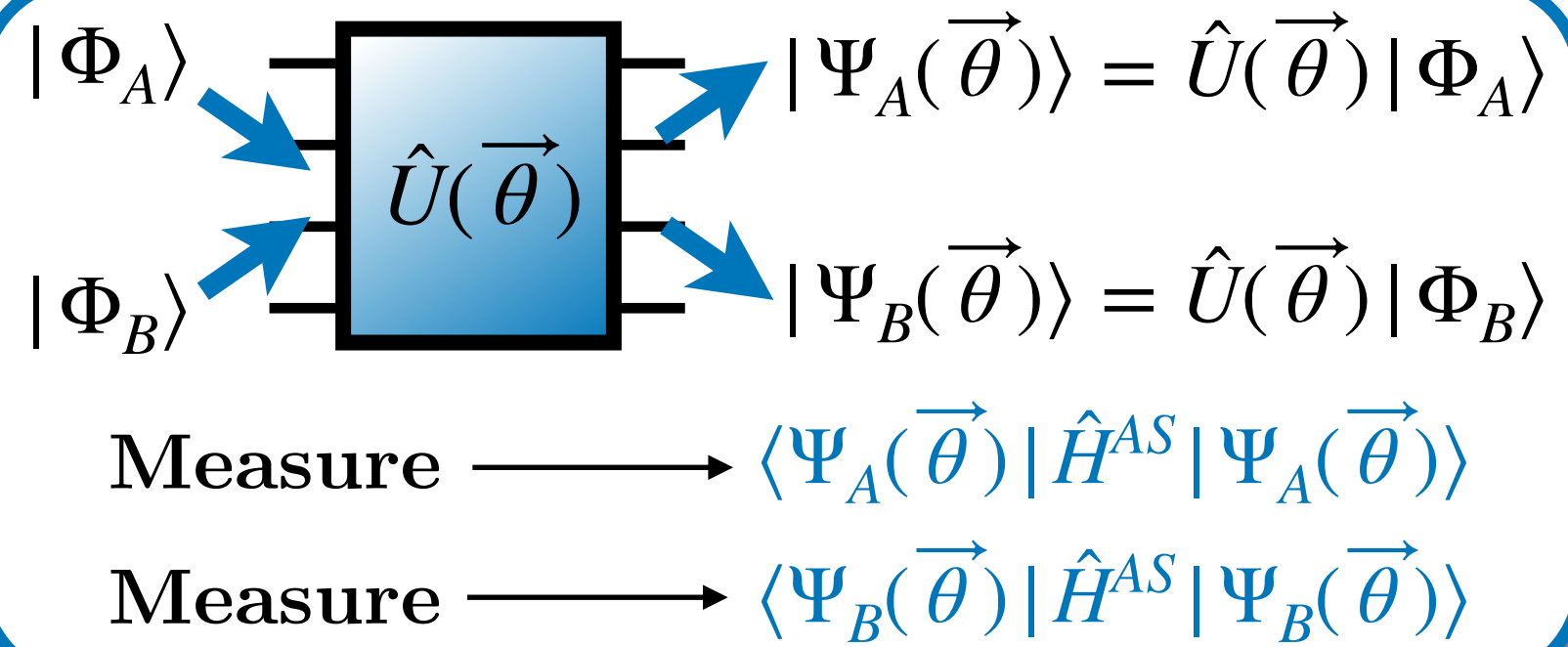
*Molecular system*



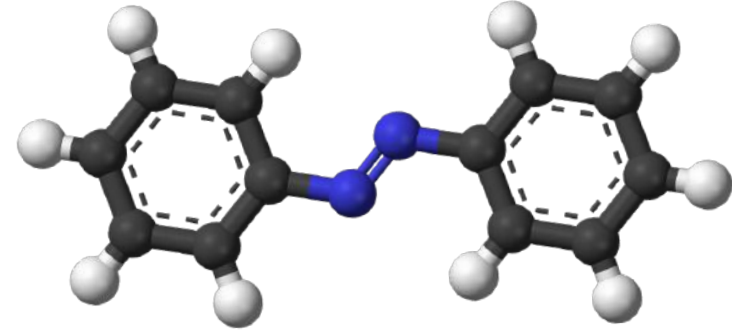
$\hat{H}^{AS}$

# III) Quantum algorithm for photochemistry

## State-Averaged VQE



*Molecular system*



$\hat{H}^{AS}$



# III) Quantum algorithm for photochemistry

## State-Averaged VQE

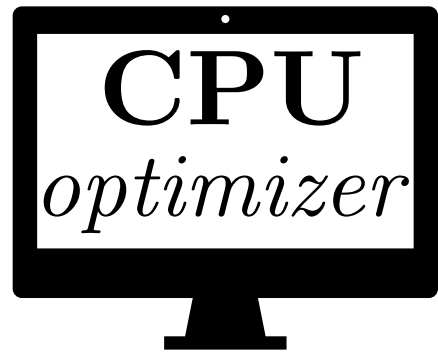
$$|\Phi_A\rangle \rightarrow \hat{U}(\vec{\theta}) \rightarrow |\Psi_A(\vec{\theta})\rangle = \hat{U}(\vec{\theta})|\Phi_A\rangle$$
$$|\Phi_B\rangle \rightarrow \hat{U}(\vec{\theta}) \rightarrow |\Psi_B(\vec{\theta})\rangle = \hat{U}(\vec{\theta})|\Phi_B\rangle$$

Measure  $\rightarrow \langle \Psi_A(\vec{\theta}) | \hat{H}^{AS} | \Psi_A(\vec{\theta}) \rangle$

Measure  $\rightarrow \langle \Psi_B(\vec{\theta}) | \hat{H}^{AS} | \Psi_B(\vec{\theta}) \rangle$

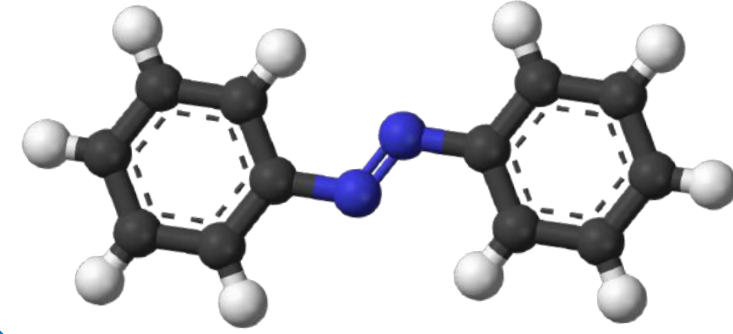


State-averaged energy cost function



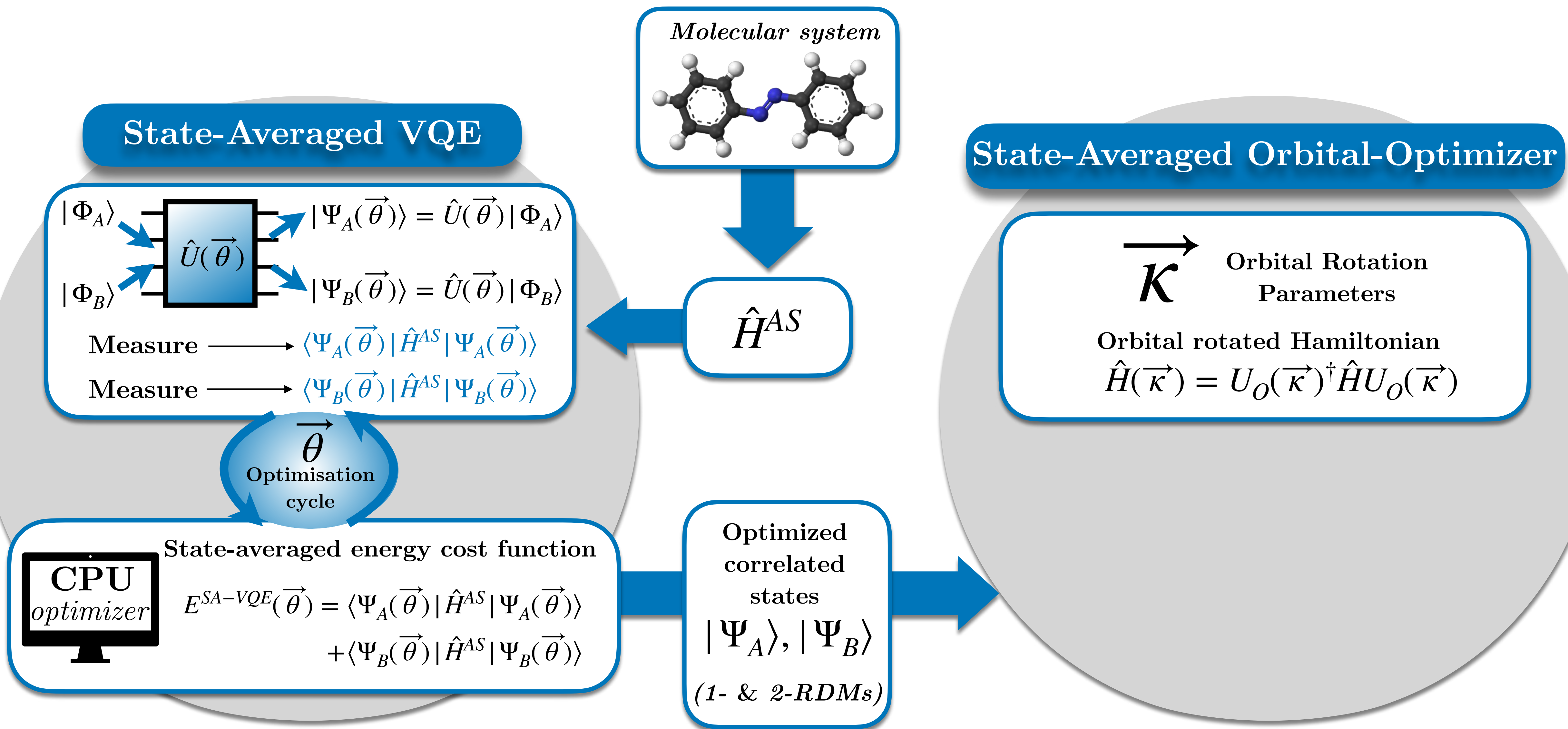
$$E^{SA-VQE}(\vec{\theta}) = \langle \Psi_A(\vec{\theta}) | \hat{H}^{AS} | \Psi_A(\vec{\theta}) \rangle + \langle \Psi_B(\vec{\theta}) | \hat{H}^{AS} | \Psi_B(\vec{\theta}) \rangle$$

*Molecular system*

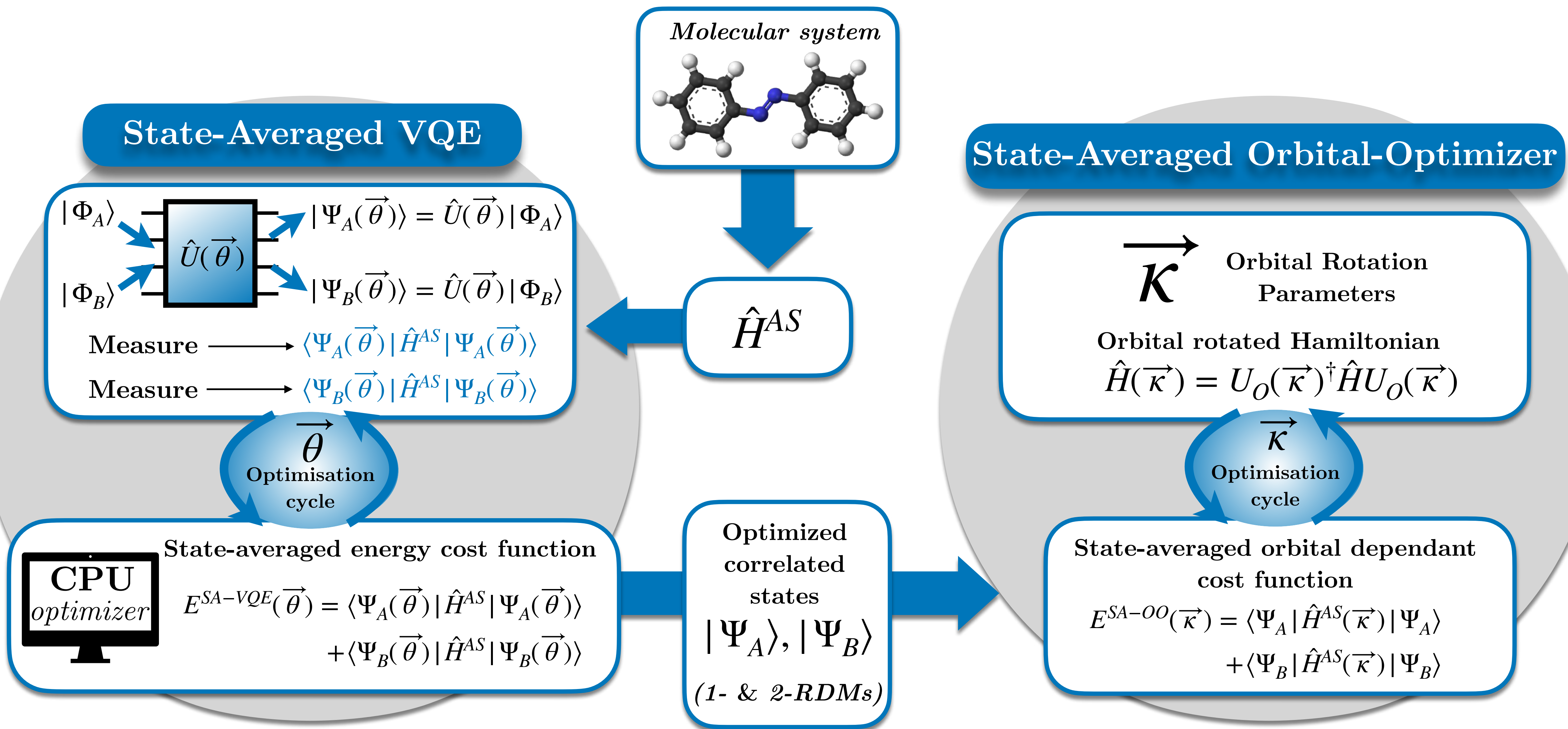


$\hat{H}^{AS}$

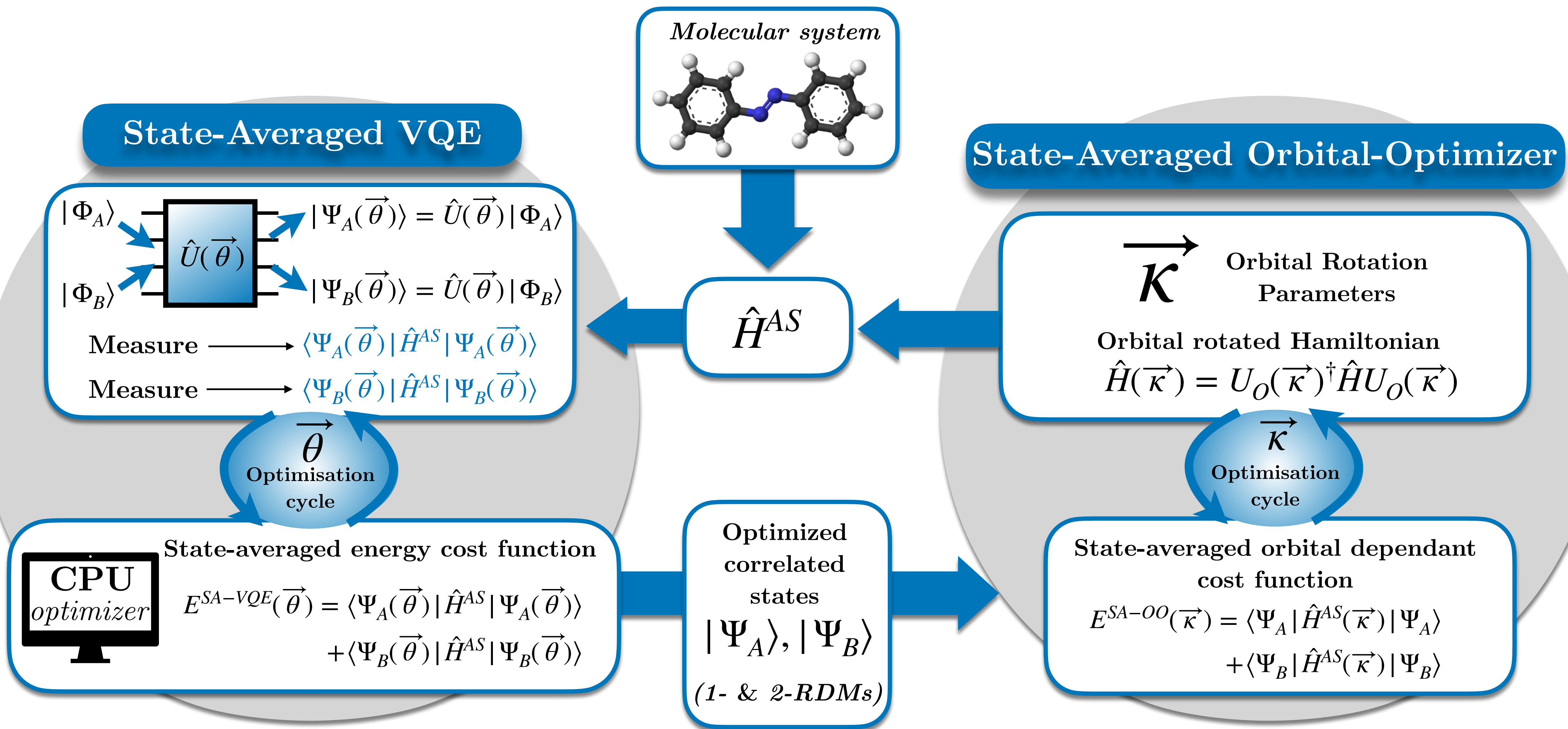
# III) Quantum algorithm for photochemistry



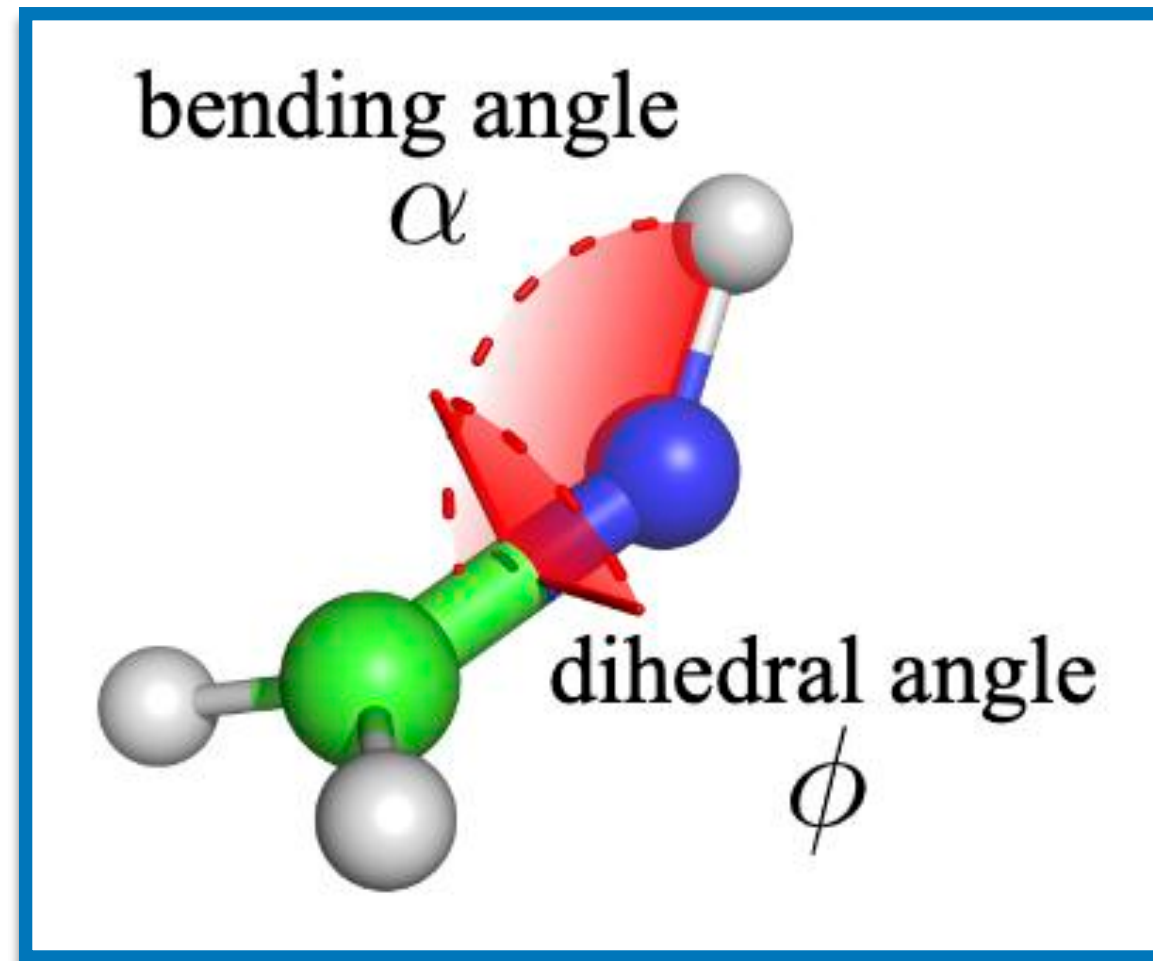
# III) Quantum algorithm for photochemistry



# III) Quantum algorithm for photochemistry



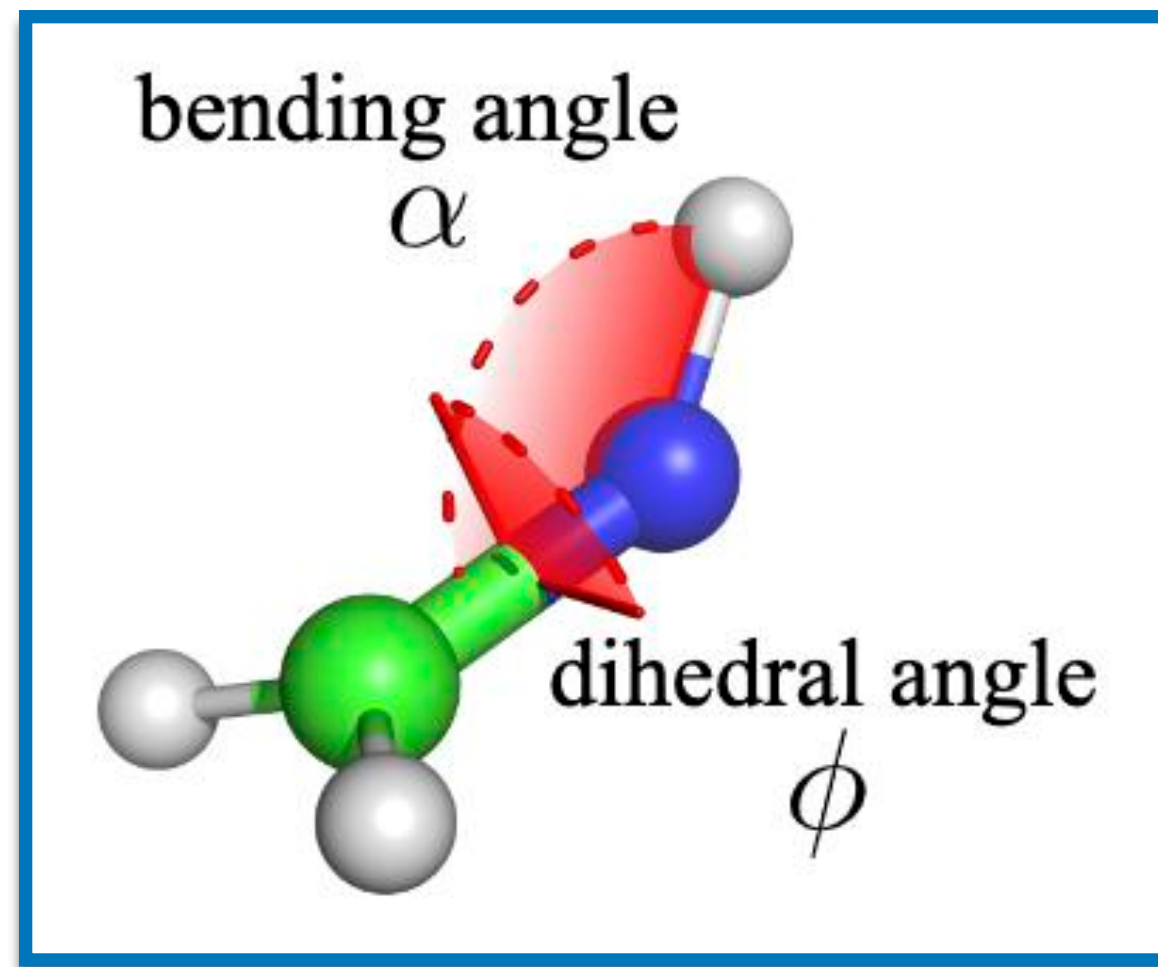
# III) Quantum algorithm for photochemistry



## Setup :

- ▶ cc-pVDZ basis
- ▶ Active space (4 elec. in 3 orb.)
- ▶ Optimiser = SLSQP
- ▶ Generalised UCCD ansatz

# III) Quantum algorithm for photochemistry

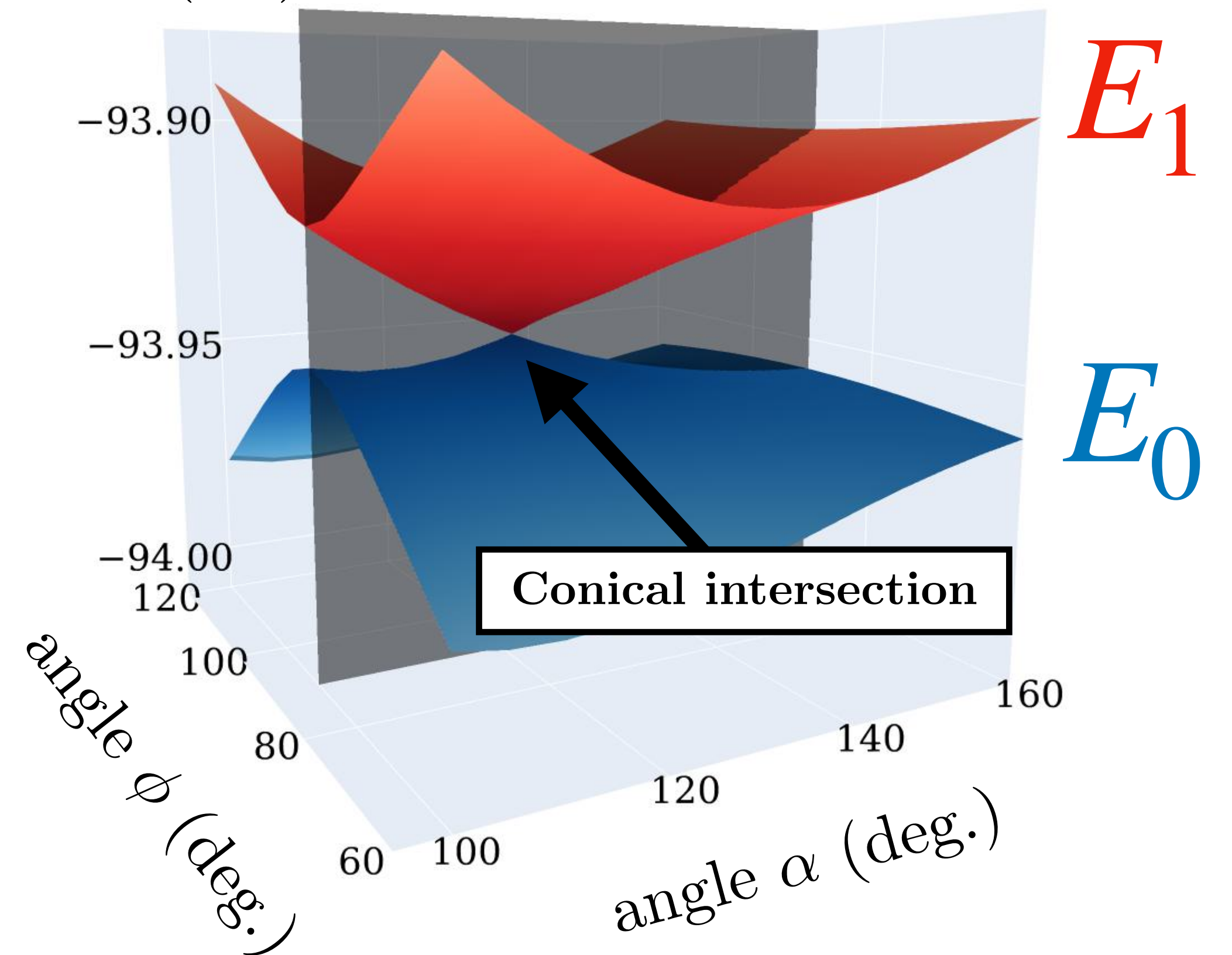


SA-OO-VQE  
Noiseless  
Simulations

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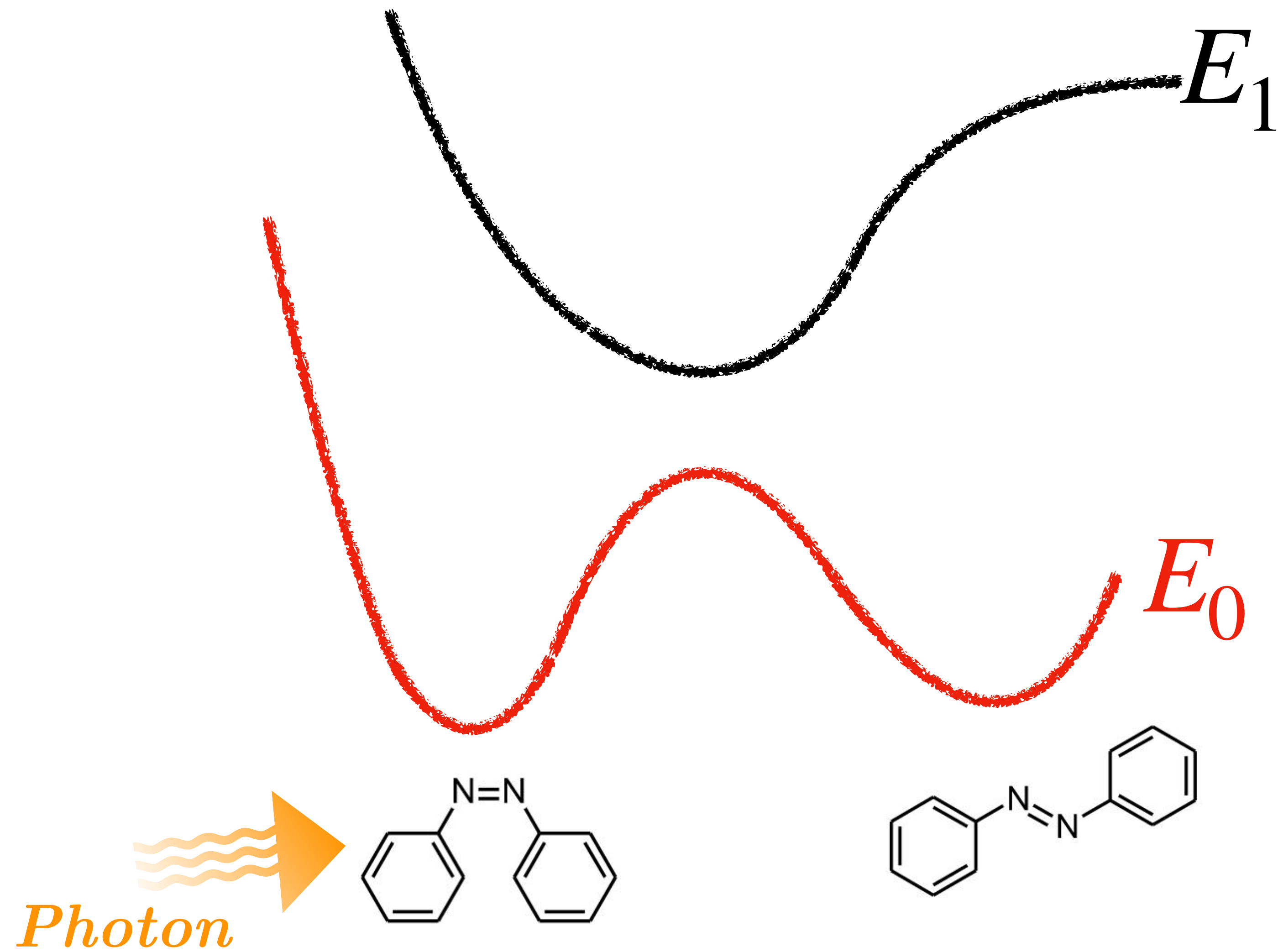
Energy (Ha)



*Ground and first excited state PESs*

# III) Quantum algorithm for photochemistry

*Ab initio Quantum Dynamics*



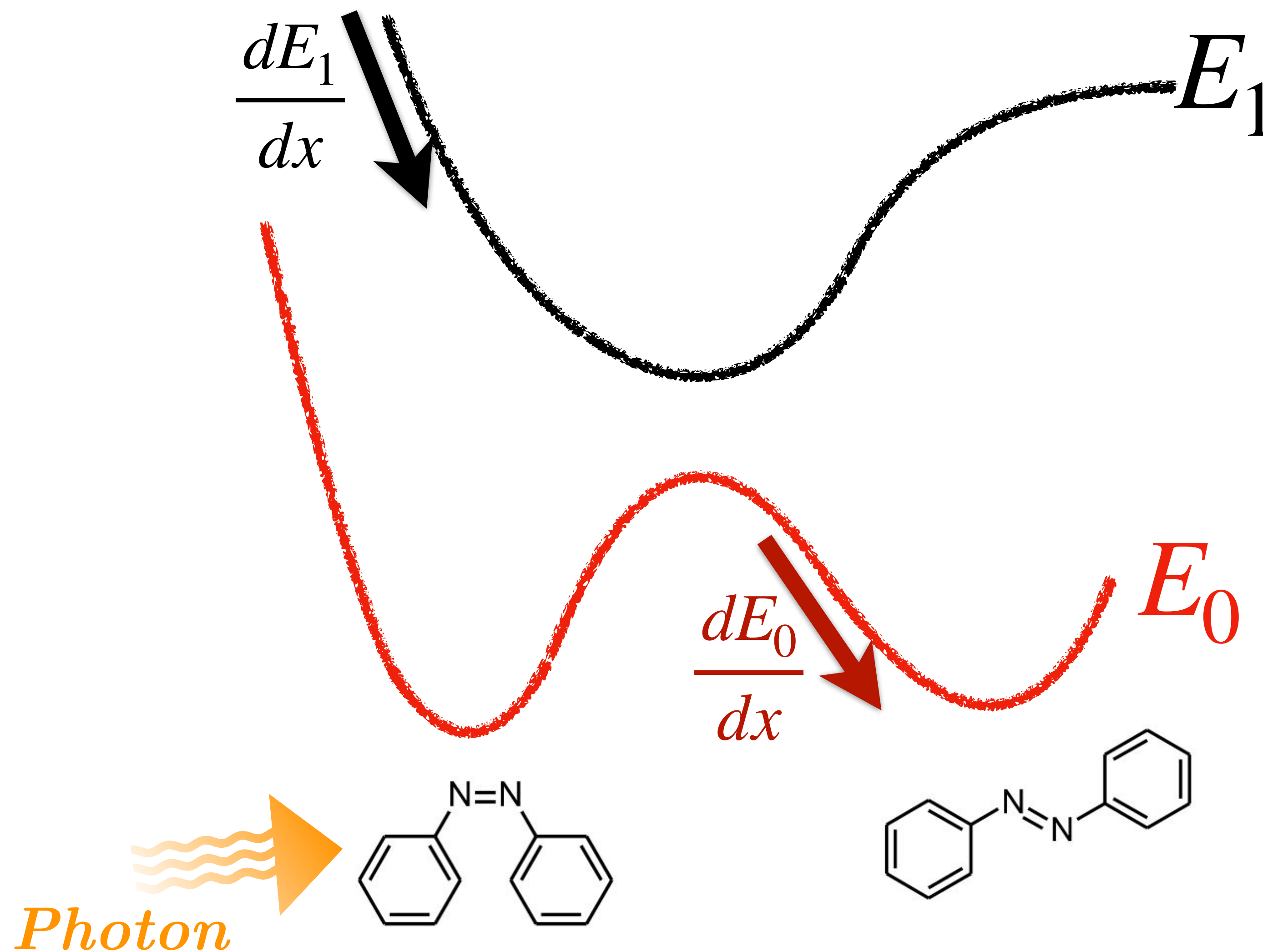
# III) Quantum algorithm for photochemistry

## Ab initio Quantum Dynamics

Nuclear derivatives

$$\frac{dE_I}{dx}$$

Nuclear forces with respect to coordinate "x"





# III) Quantum algorithm for photochemistry

## Ab initio Quantum Dynamics

Nuclear derivatives

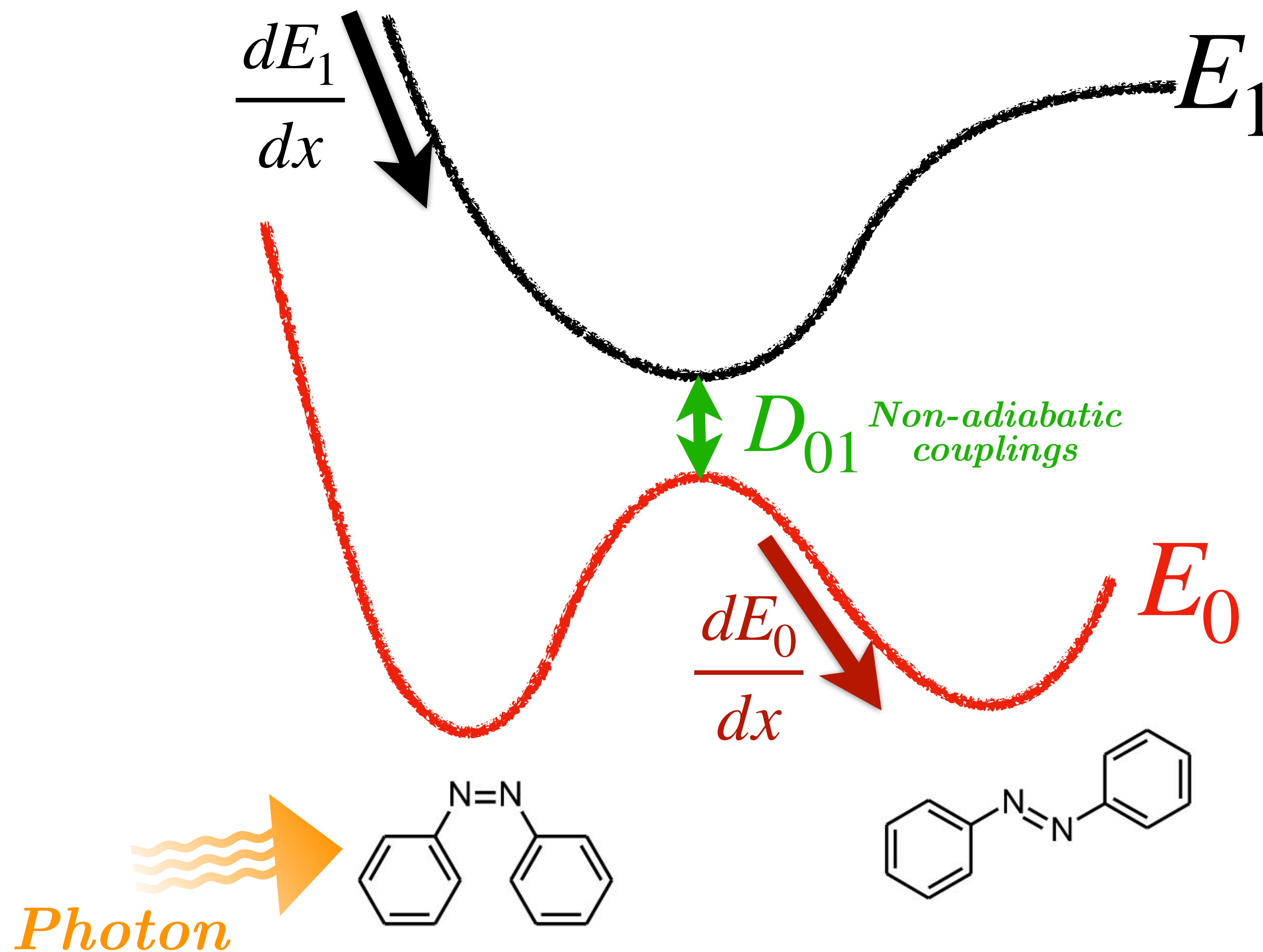
$$\frac{dE_I}{dx}$$

Nuclear forces with respect to coordinate “  $x$  ”

Non-adiabatic couplings

$$D_{IJ} = \langle \Psi_I | \frac{d}{dx} \Psi_J \rangle$$

Coupling between two states through nuclear vibrations



# III) Quantum algorithm for photochemistry

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$$\frac{dE_I}{dx} \xrightarrow{\text{PROBLEM!}} \frac{\partial E_I}{\partial \kappa_{pq}} \neq 0 \quad \& \quad \frac{\partial E_I}{\partial \theta_n} \neq 0$$

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Lagrange multiplier method

$$\mathcal{L}_I = E_I + \sum_{pq} \bar{\kappa}_{pq}^I \frac{\partial E^{SA}}{\partial \kappa_{pq}} + \sum_n \bar{\theta}_n^I \frac{\partial E^{SA}}{\partial \theta_n}$$

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**Can be measured out of the circuit !**

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**Final analytical form**

$$\begin{pmatrix} \mathbf{H}_{SA}^{OO} & \mathbf{H}_{SA}^{OC} \\ \mathbf{H}_{SA}^{CO} & \mathbf{H}_{SA}^{CC} \end{pmatrix} \begin{pmatrix} \bar{\kappa}^I \\ \bar{\theta}^I \end{pmatrix} = - \begin{pmatrix} \mathbf{G}^{O,I} \\ \mathbf{G}^{C,I} \end{pmatrix}$$

**Can be measured out of the circuit !**

$$\frac{dE_I}{dx} = \sum_{pq} \frac{\partial h_{pq}}{\partial x} \gamma_{pq}^{I,eff} + \frac{1}{2} \sum_{pqrs} \frac{\partial g_{pqrs}}{\partial x} \Gamma_{pqrs}^{I,eff} + \sum_J \sum_n w_J \bar{\theta}_n^I G_n^{C,J} \left( \frac{\partial \hat{H}}{\partial x} \right)$$

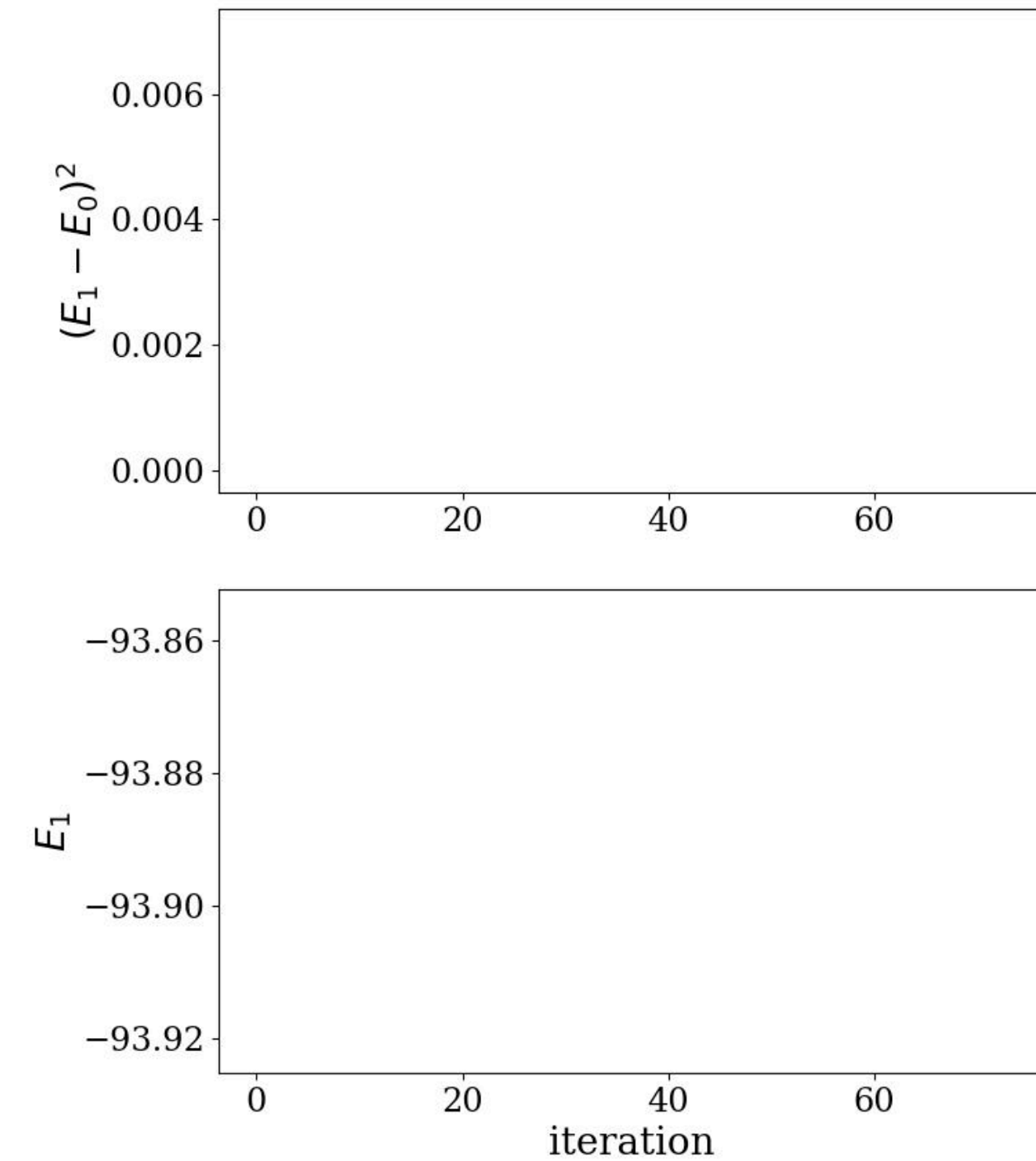
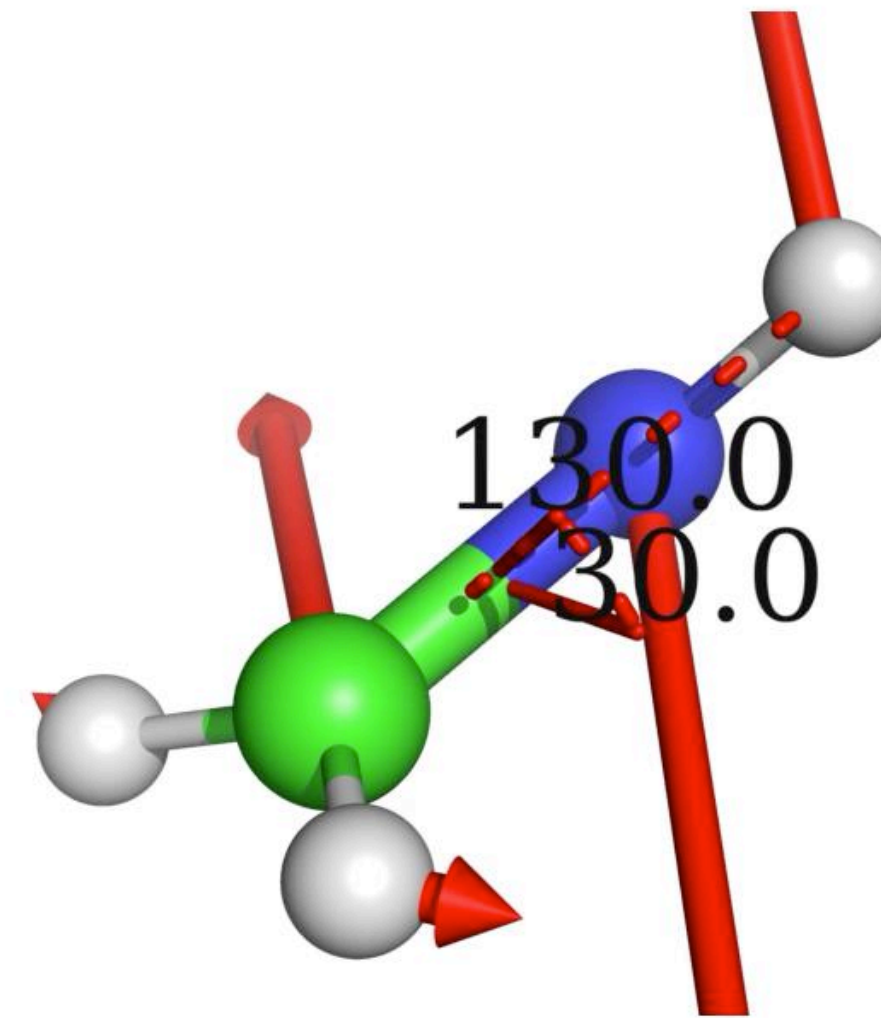
# III) Quantum algorithm for photochemistry

*Ingredients:*

- Nuclear gradients
- Non-adiabatic couplings

Research of  
Minimal-energy  
conical-intersection  
(MECI)

Noiseless Simulations



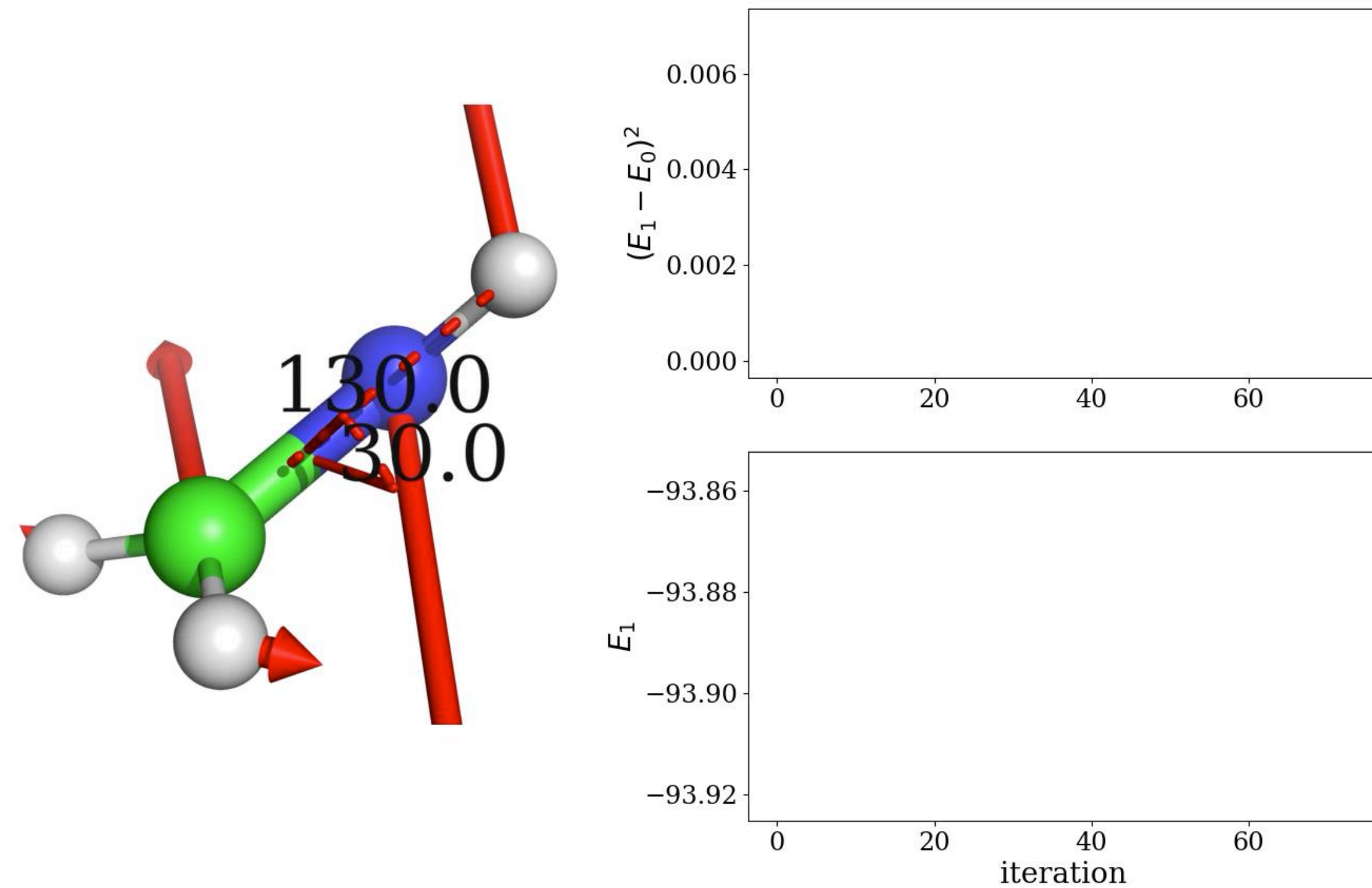
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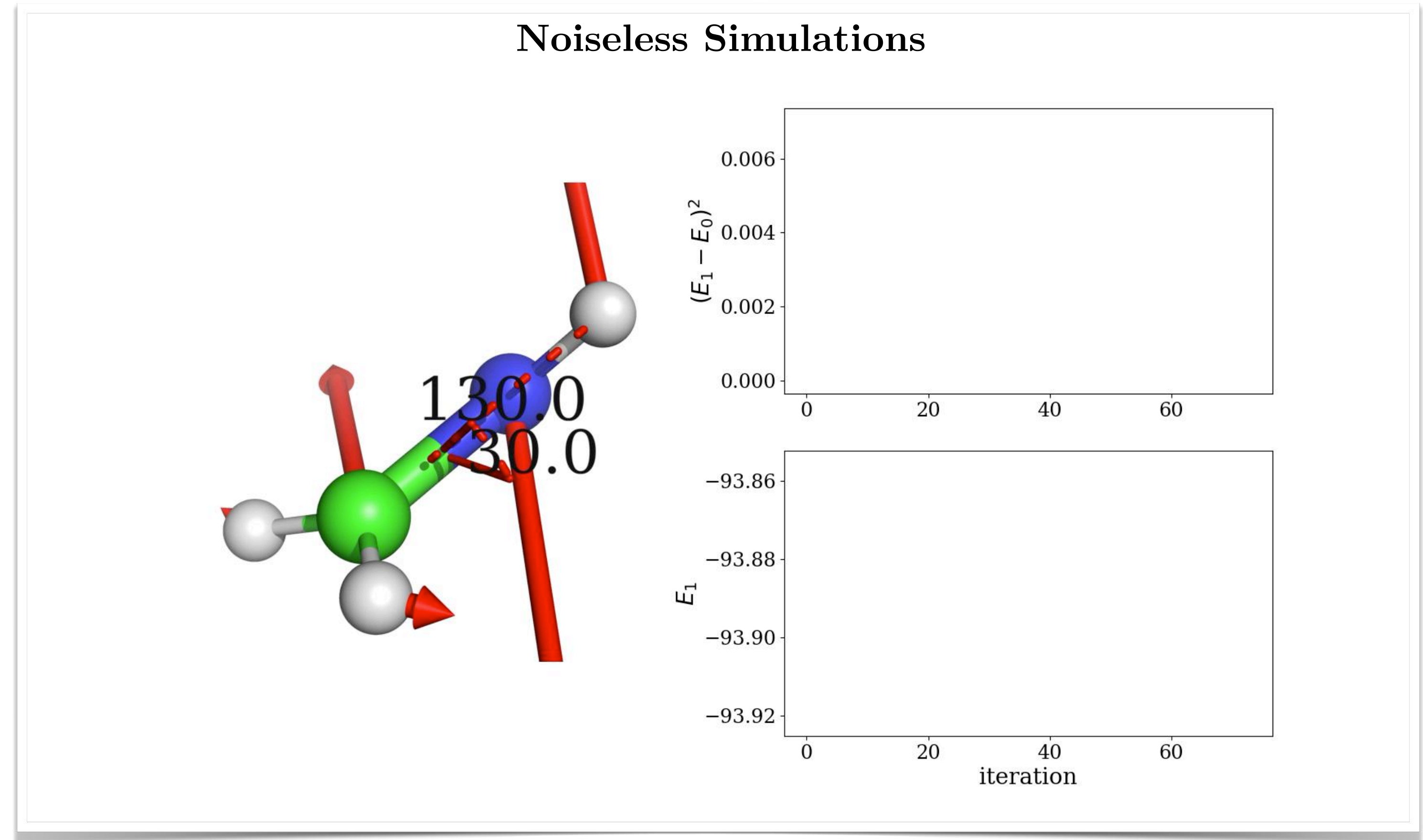


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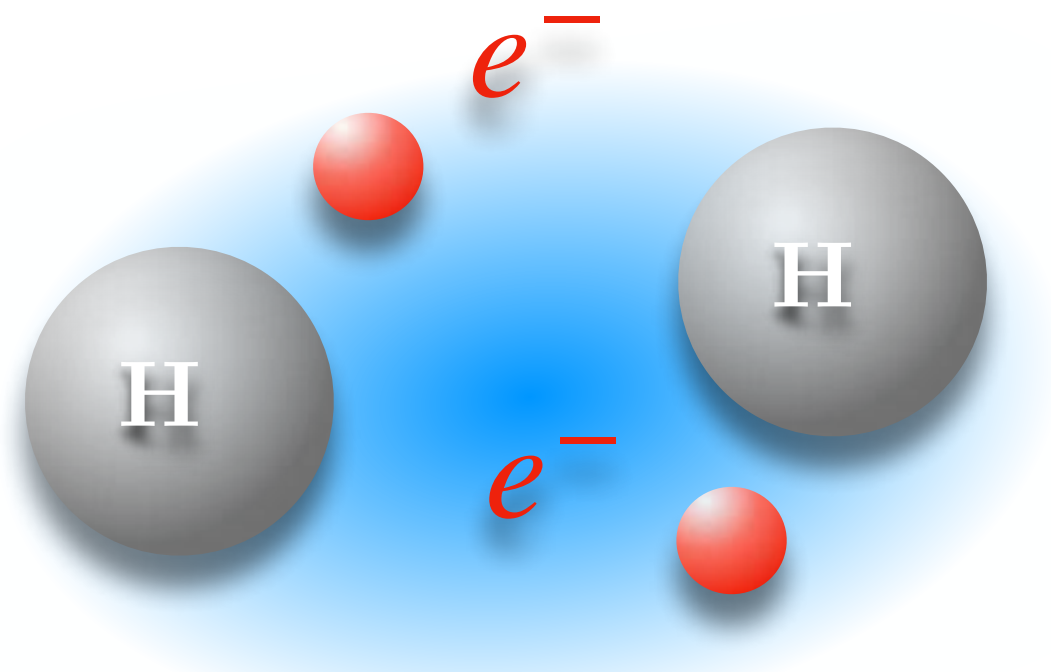
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Minimal-energy  
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*SA-OO-VQE = Quantum analog of SA-CASSCF*

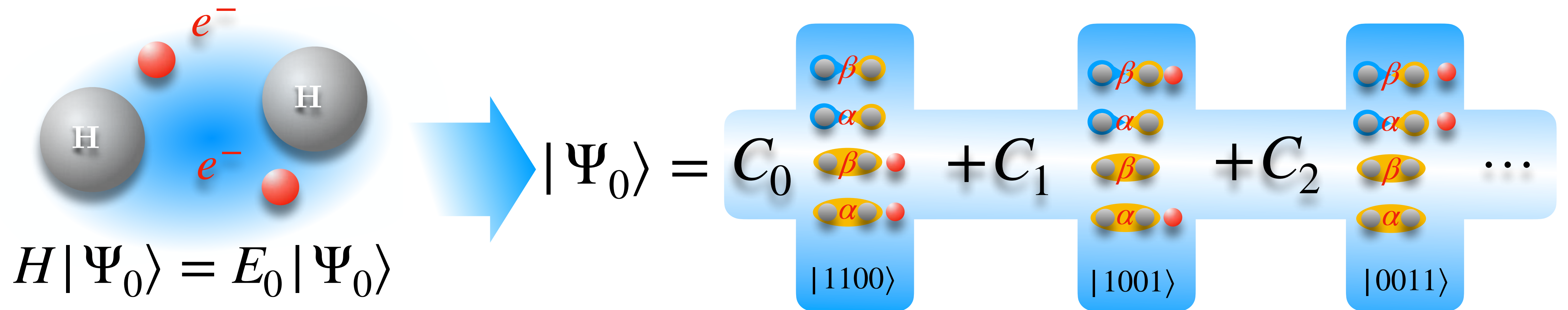
# Take Home Messages

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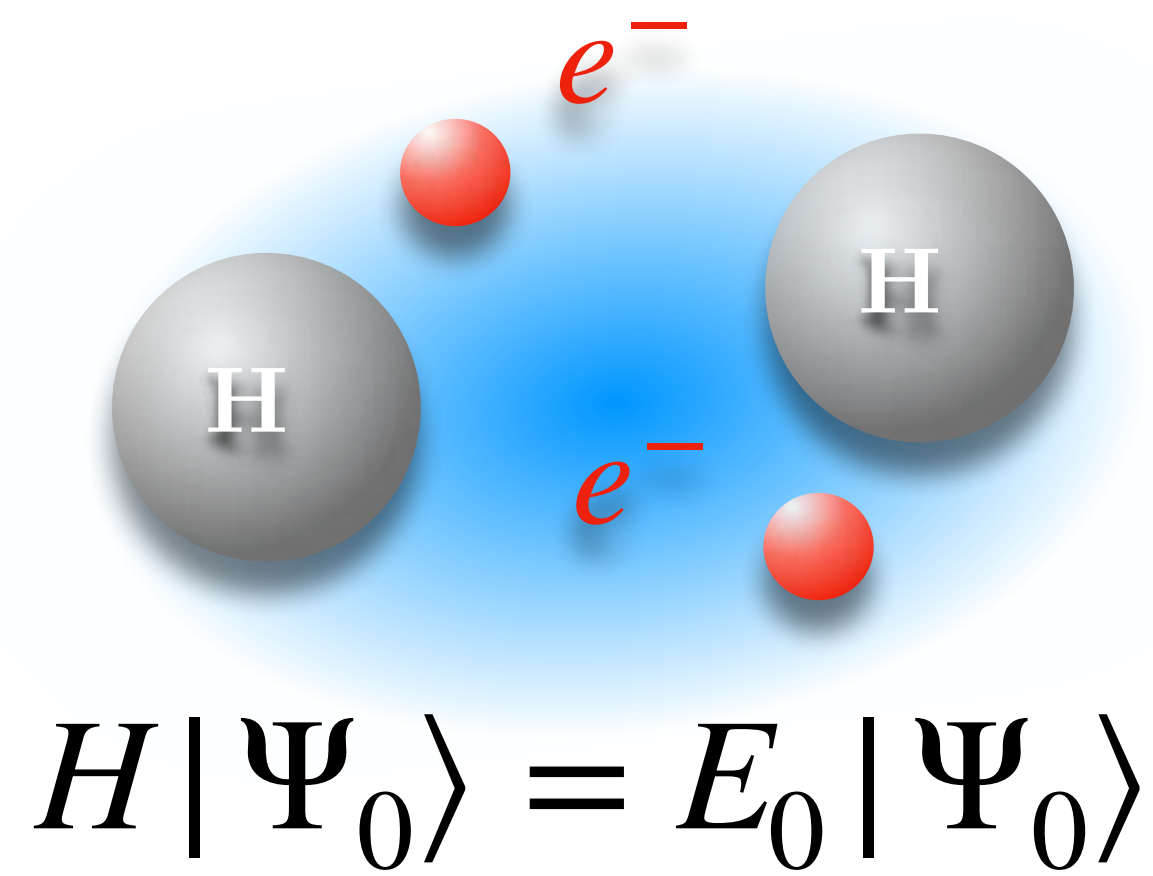


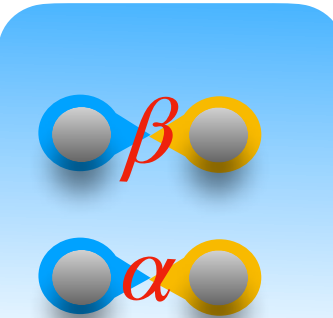
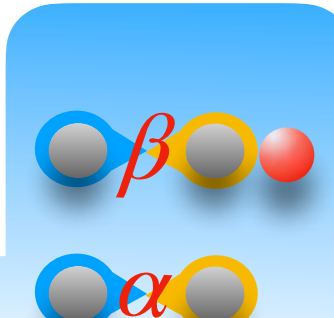
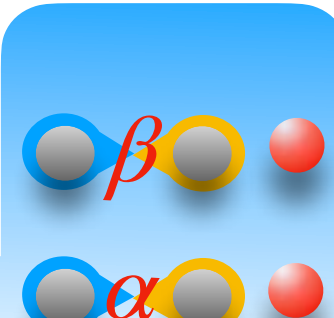
$$H|\Psi_0\rangle = E_0|\Psi_0\rangle$$

# Take Home Messages



# Take Home Messages



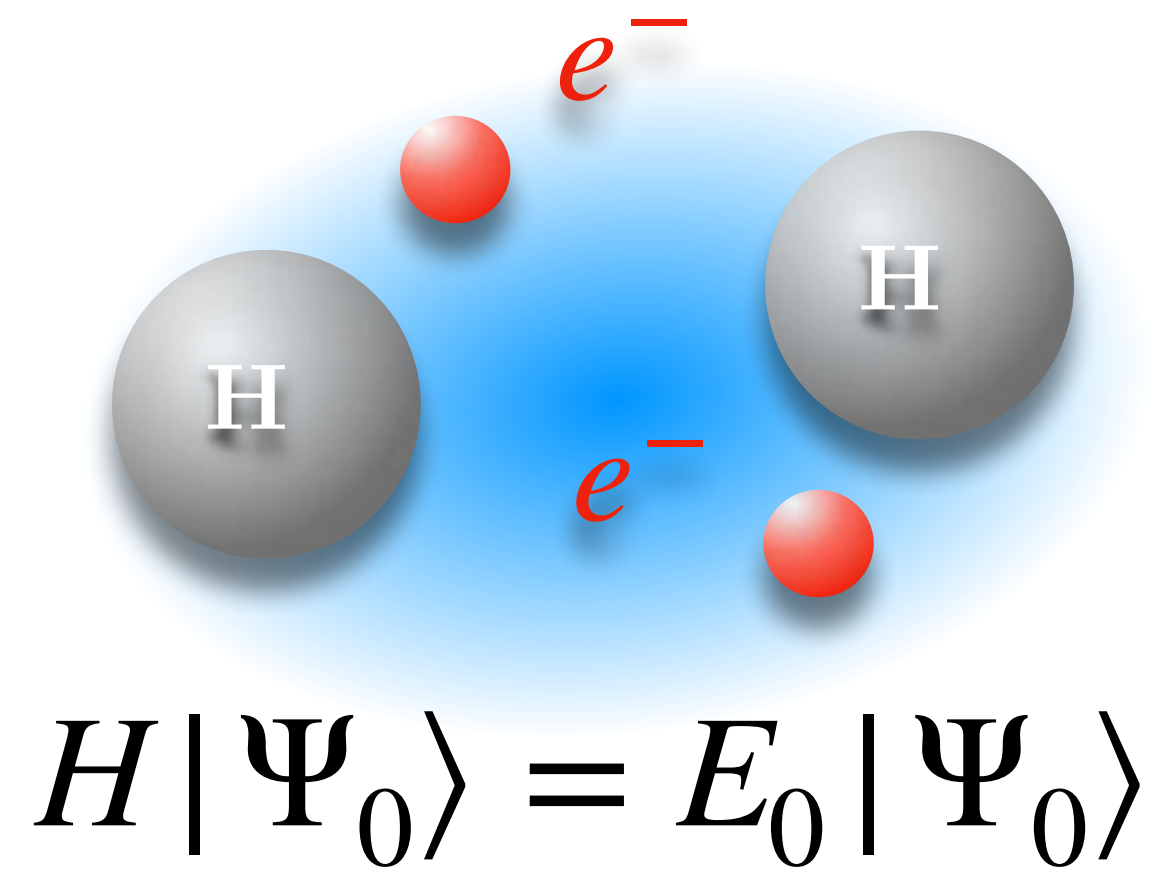
$|\Psi_0\rangle = C_0$    $+ C_1$    $+ C_2$    $\dots$

$|1100\rangle$   $|1001\rangle$   $|0011\rangle$

Quantum Algorithm



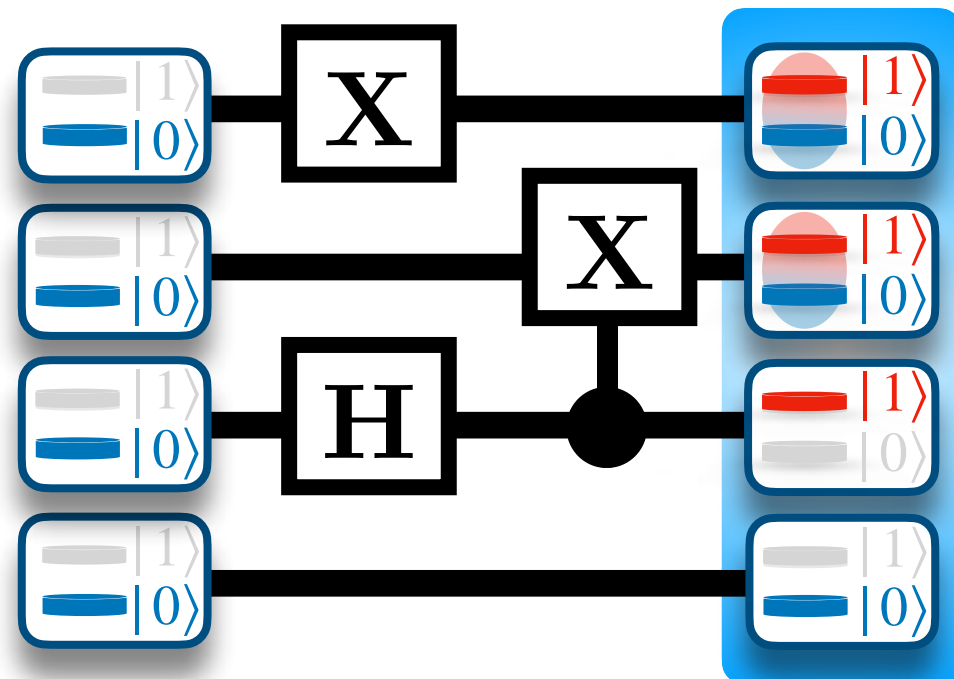
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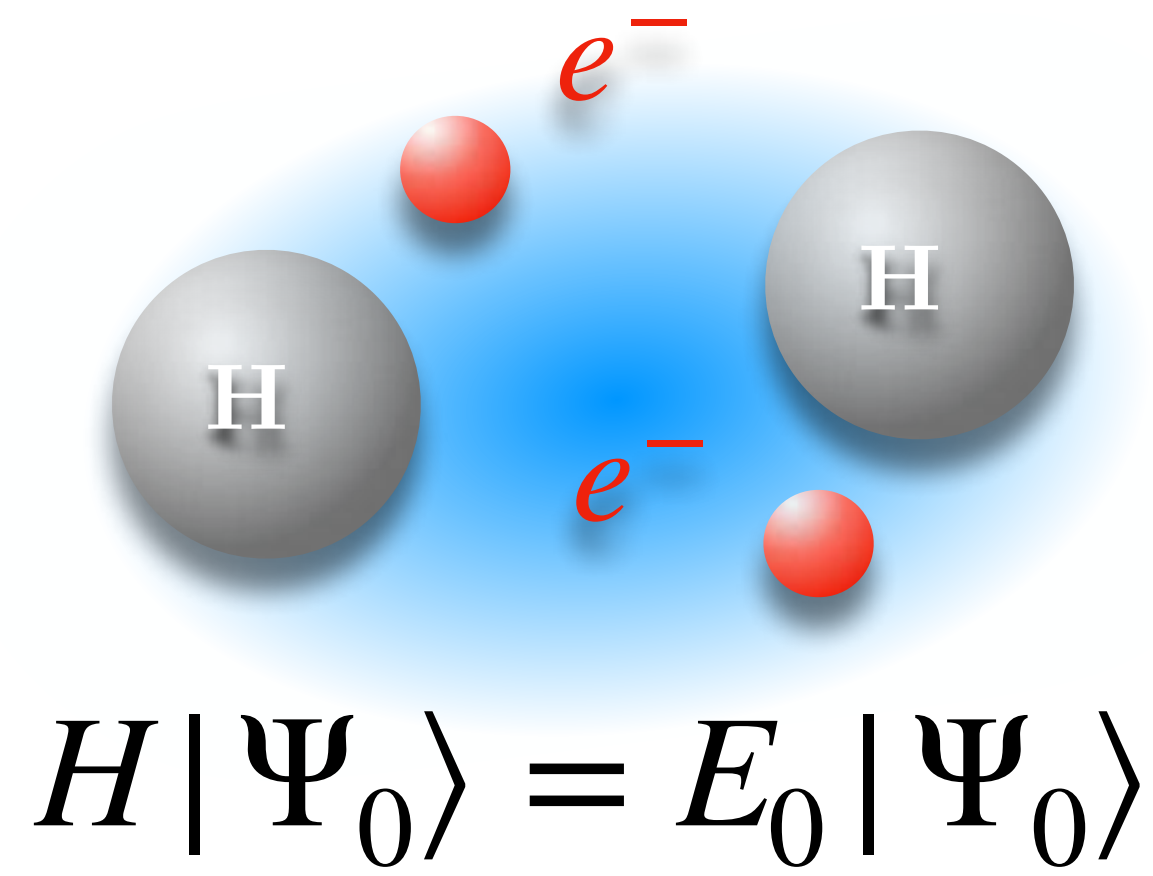
$$|\Psi_0\rangle = C_0 \begin{array}{c} \beta \\ \alpha \\ \beta \\ \alpha \end{array} + C_1 \begin{array}{c} \beta \\ \alpha \\ \beta \\ \alpha \end{array} + C_2 \begin{array}{c} \beta \\ \alpha \\ \beta \\ \alpha \end{array} + \dots$$

$|1100\rangle$        $|1001\rangle$        $|0011\rangle$

## Quantum Algorithm



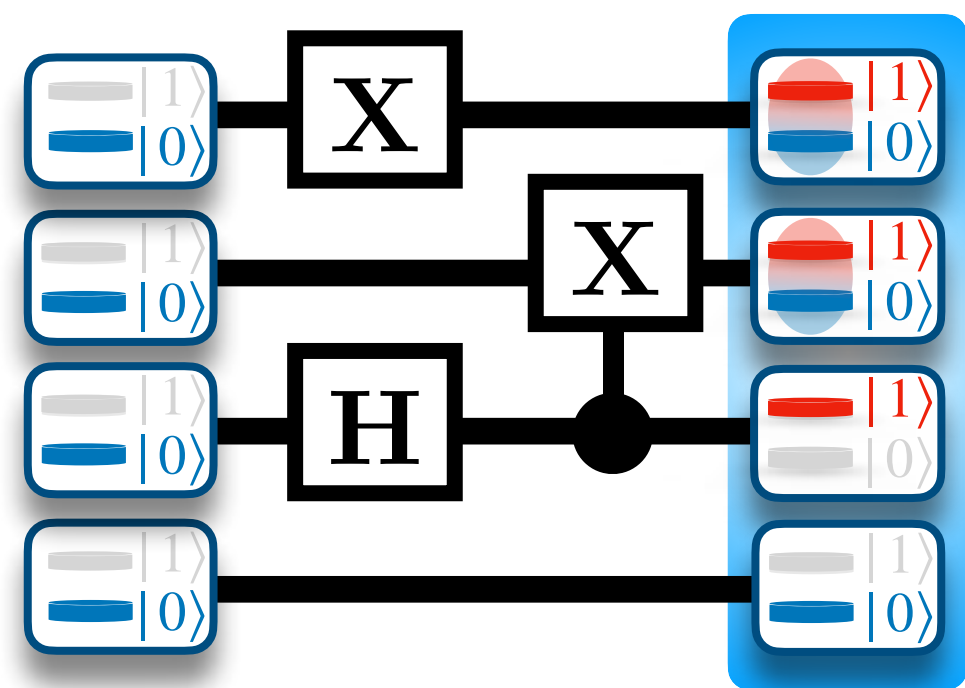
# Take Home Messages



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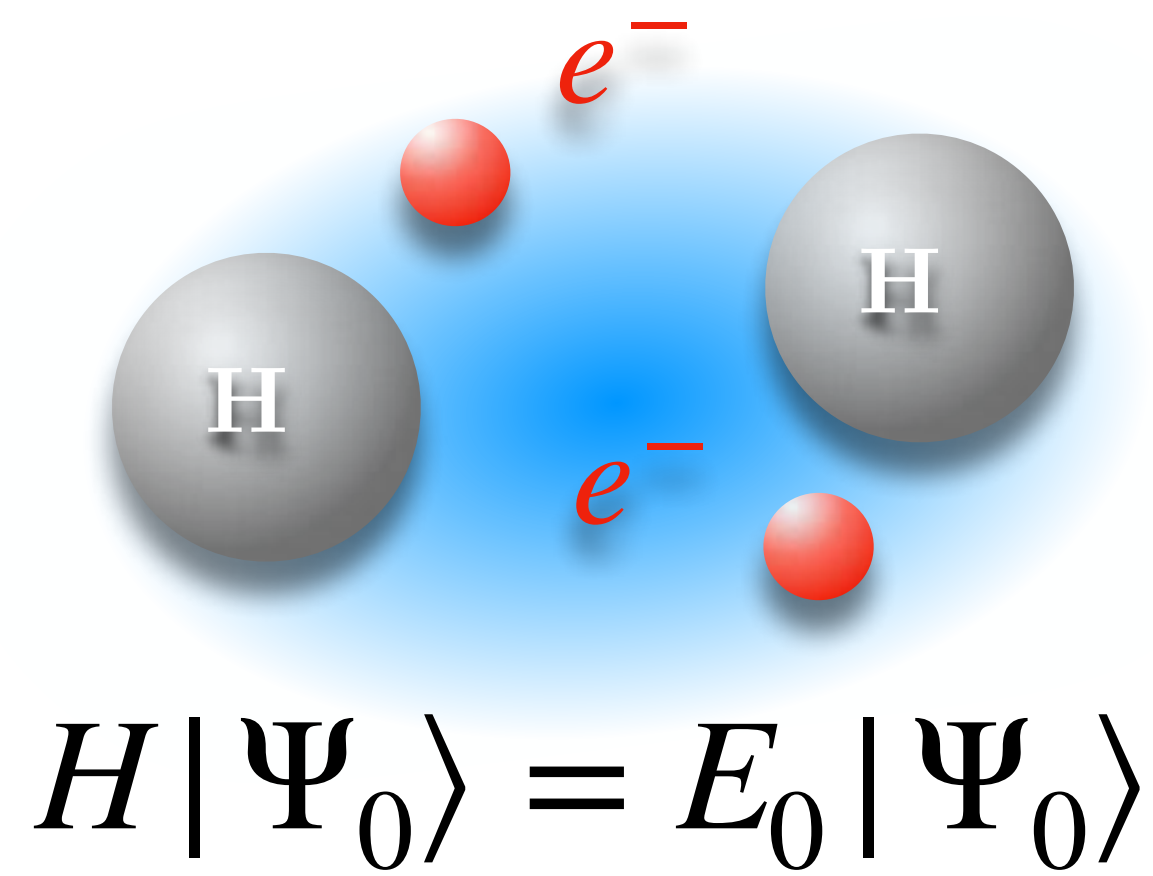
$|1100\rangle$                        $|1001\rangle$                        $|0011\rangle$

## Quantum Algorithm



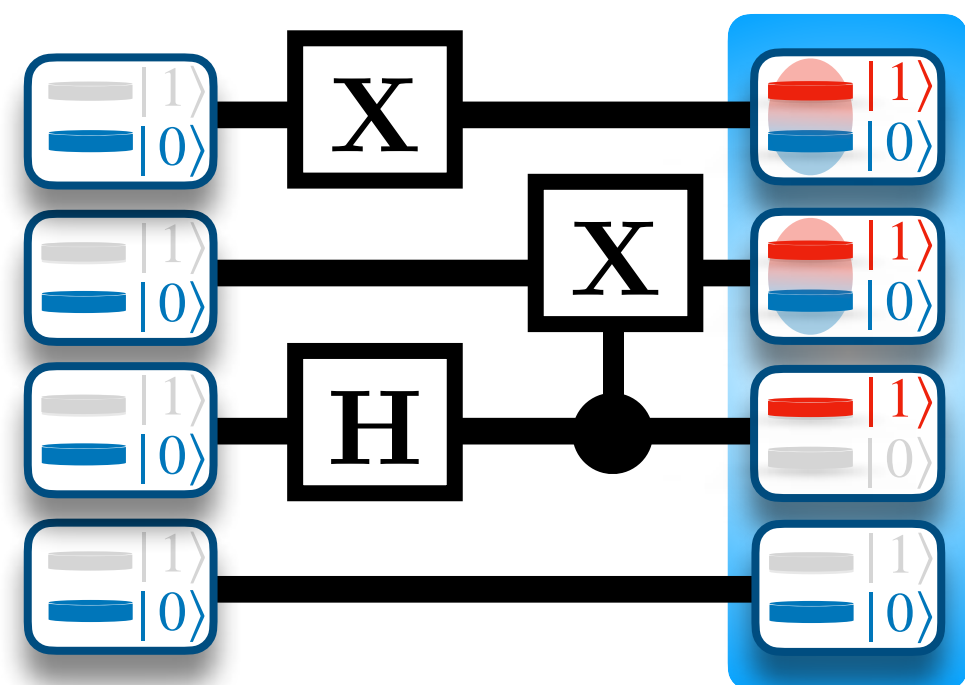
$$|\Psi_{\text{qubits}}\rangle = C_0 \begin{array}{c} |1\rangle \\ |0\rangle \\ |1\rangle \\ |0\rangle \end{array} + C_1 \begin{array}{c} |1\rangle \\ |0\rangle \\ |1\rangle \\ |0\rangle \end{array} + C_2 \begin{array}{c} |1\rangle \\ |0\rangle \\ |1\rangle \\ |0\rangle \end{array} + \dots$$

# Take Home Messages



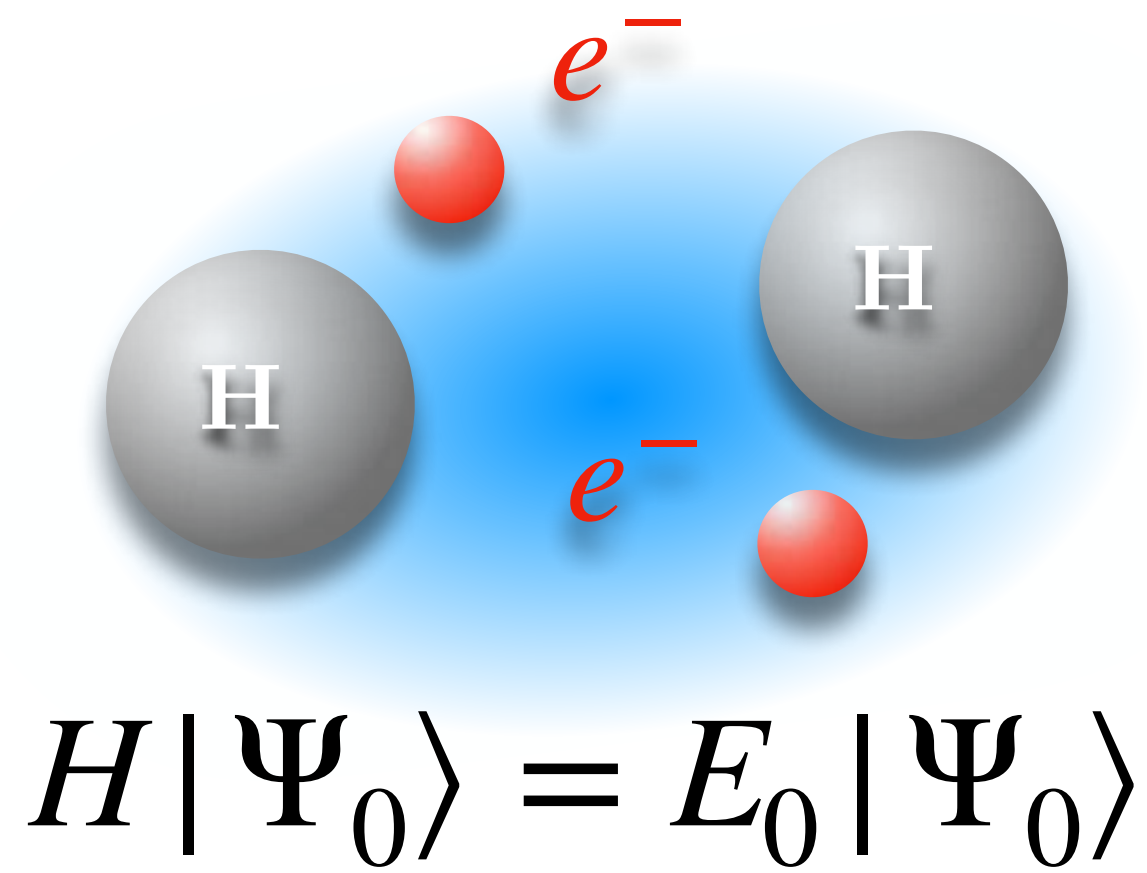
$$|\Psi_0\rangle = C_0 \begin{array}{c} \beta \\ \alpha \\ \beta \\ \alpha \end{array} |1100\rangle + C_1 \begin{array}{c} \beta \\ \alpha \\ \beta \\ \alpha \end{array} |1001\rangle + C_2 \begin{array}{c} \beta \\ \alpha \\ \beta \\ \alpha \end{array} |0011\rangle + \dots$$

## Quantum Algorithm



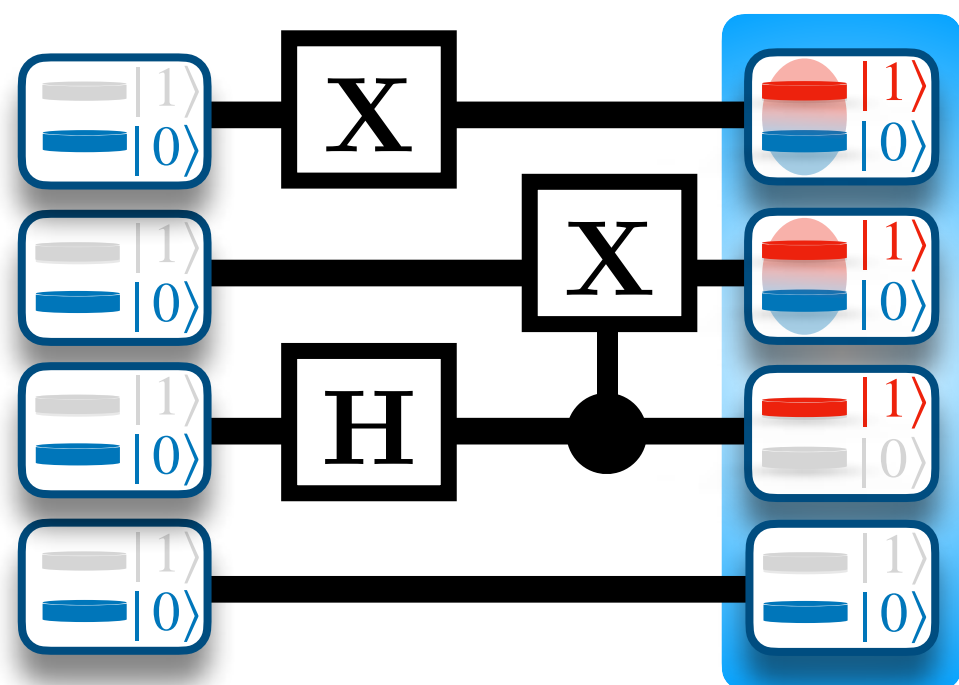
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# Take Home Messages



$$|\Psi_0\rangle = C_0 \begin{array}{c} \beta \\ \alpha \\ \beta \\ \alpha \end{array} |1100\rangle + C_1 \begin{array}{c} \beta \\ \alpha \\ \beta \\ \alpha \end{array} |1001\rangle + C_2 \begin{array}{c} \beta \\ \alpha \\ \beta \\ \alpha \end{array} |0011\rangle + \dots$$

## Quantum Algorithm



$$|\psi_{\text{qubits}}\rangle = C_0 \begin{array}{c} |1\rangle \\ |0\rangle \\ |1\rangle \\ |0\rangle \end{array} + C_1 \begin{array}{c} |1\rangle \\ |0\rangle \\ |1\rangle \\ |0\rangle \end{array} + C_2 \begin{array}{c} |1\rangle \\ |0\rangle \\ |1\rangle \\ |0\rangle \end{array} + \dots \approx |\Psi_0\rangle$$

*NISQ era (resources limitations)*

# Take Home Messages

**SA-OO-VQE:** Quantum algorithm for photo-chemistry

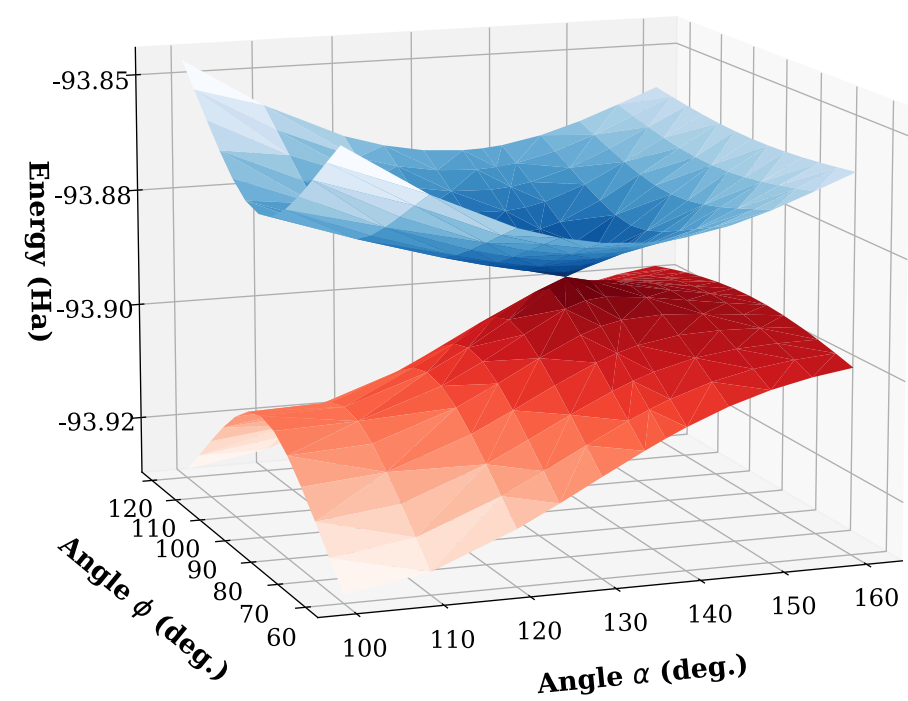
S. Yalouz et al. *Quantum Science and Technology* 6.2 (2021): 024004.

S. Yalouz et al. *Journal of chemical theory and computation* 18.2 (2022): 776-794.

# Take Home Messages

**SA-OO-VQE:** Quantum algorithm for photo-chemistry

Description of  
degenerated PES



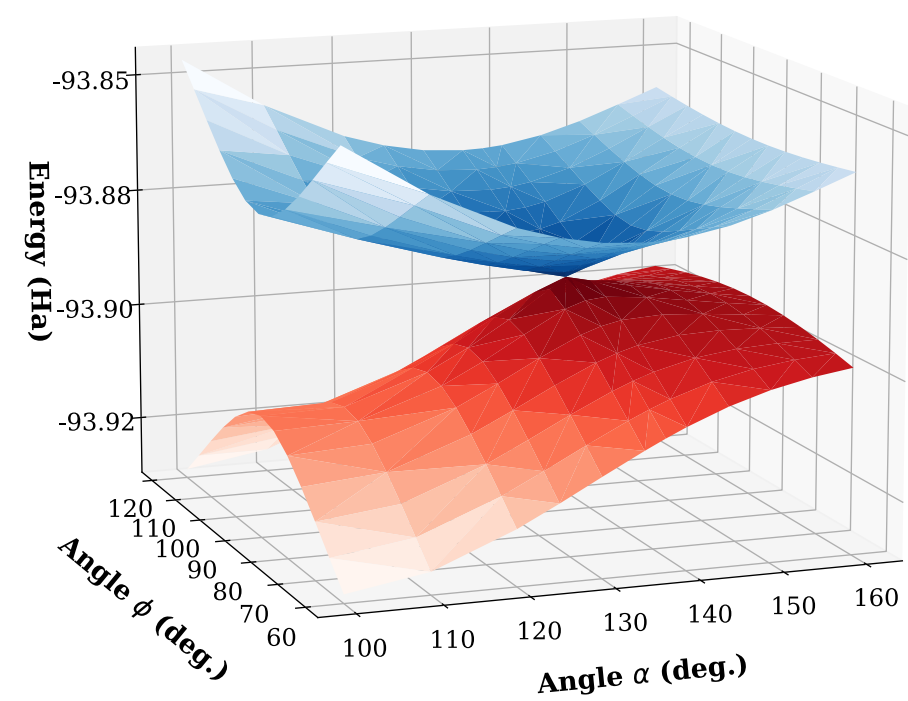
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# Take Home Messages

**SA-OO-VQE:** Quantum algorithm for photo-chemistry

Description of  
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Nuclear  
derivatives

$$\frac{dE_I}{dx}$$

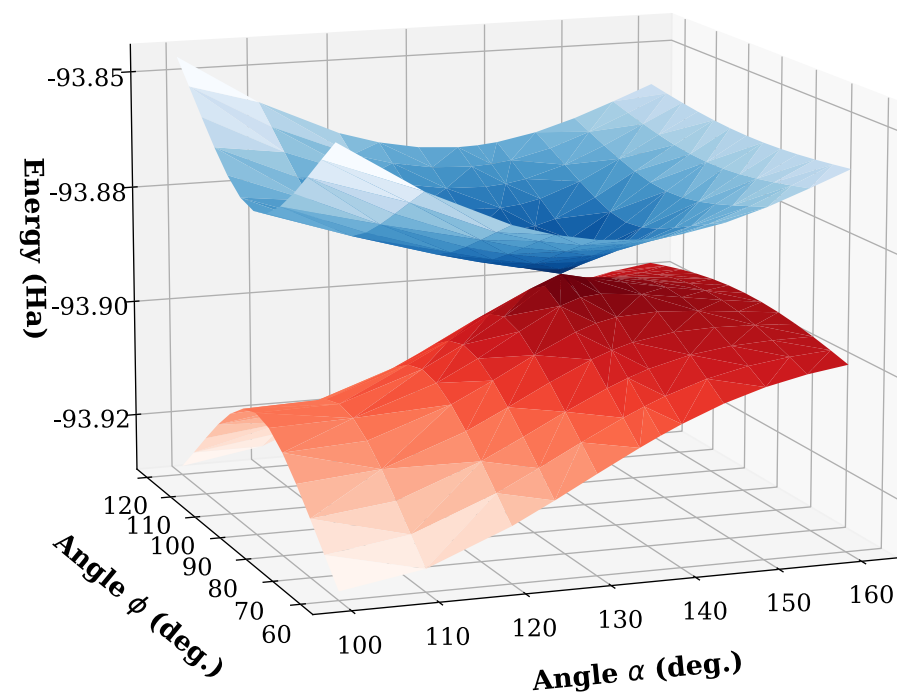
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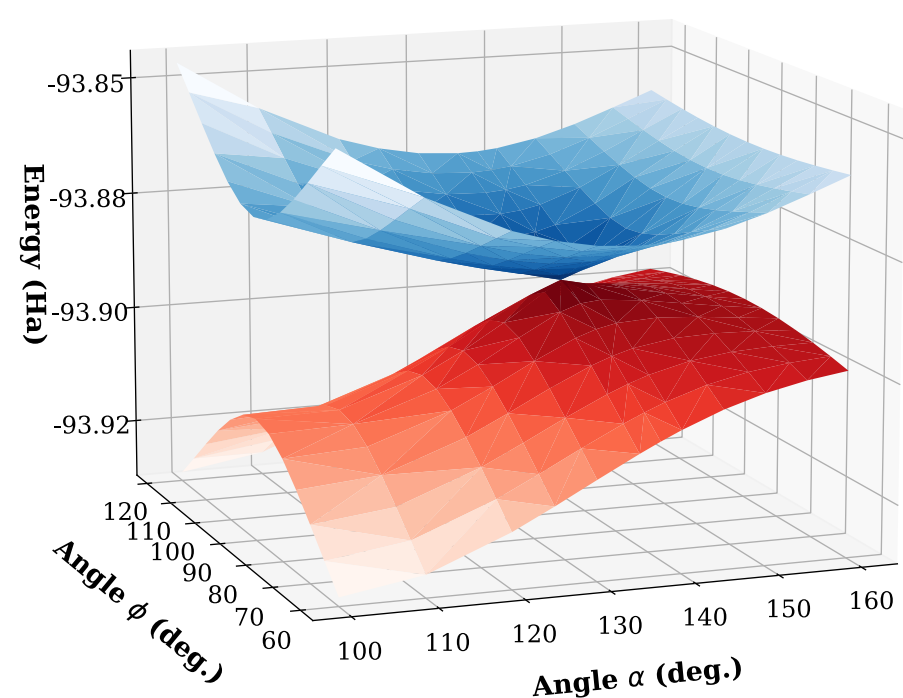
S. Yalouz et al. *Journal of chemical theory and computation* 18.2 (2022): 776-794.



# Take Home Messages

## SA-OO-VQE: Quantum algorithm for photo-chemistry

### Description of degenerated PES

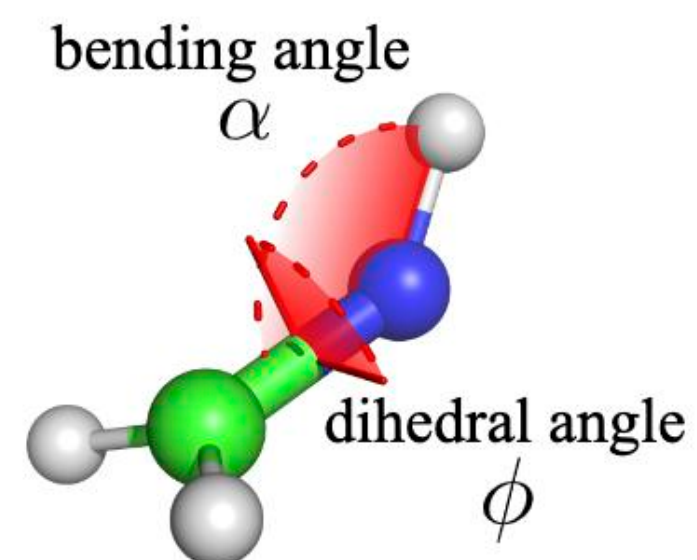


### Nuclear derivatives

$$\frac{dE_I}{dx}$$

### Non-adiabatic couplings

$$D_{IJ} = \left\langle \Psi_I \left| \frac{d}{dx} \Psi_J \right. \right\rangle$$



### MECI optimization

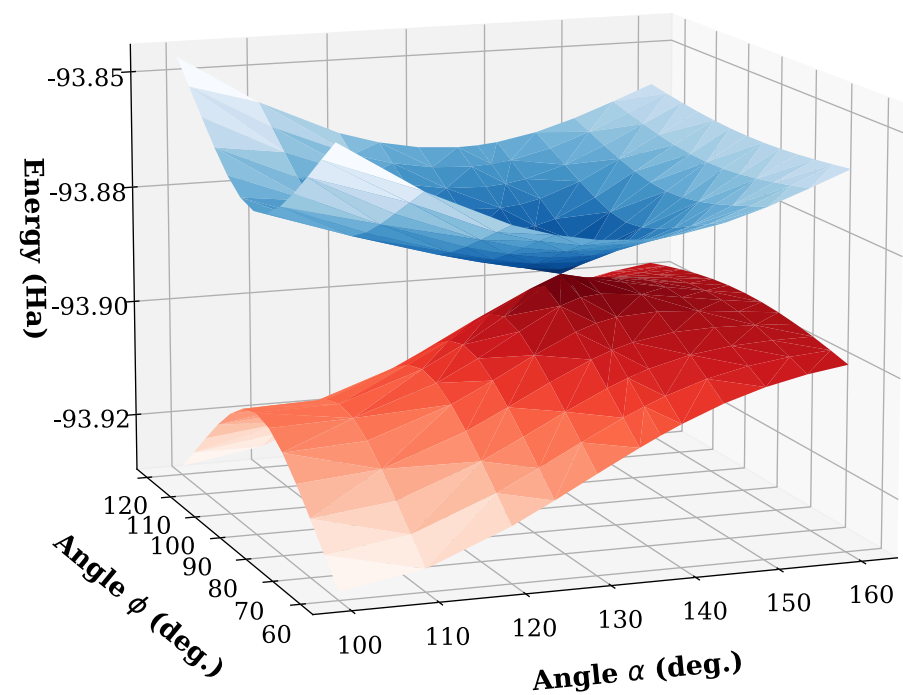
S. Yalouz et al. *Quantum Science and Technology* 6.2 (2021): 024004.

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# Take Home Messages

SA-OO-VQE: Quantum algorithm for photo-chemistry

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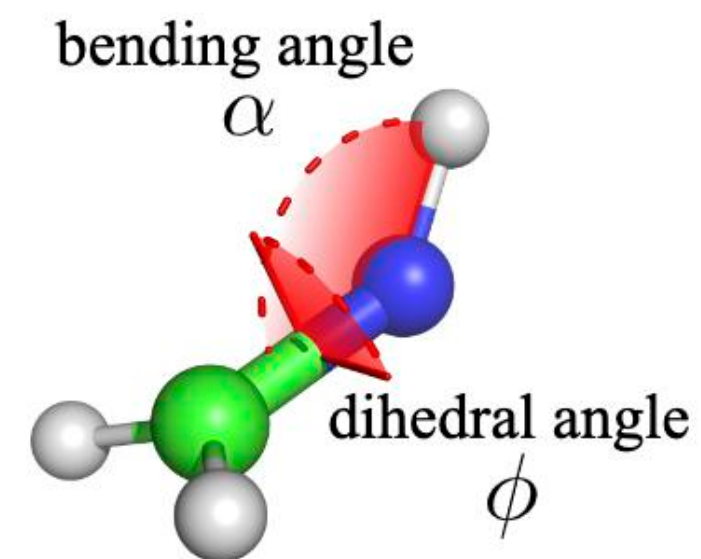


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MECI optimization

**LCQIS**  
Quantum  
Software/Hardware

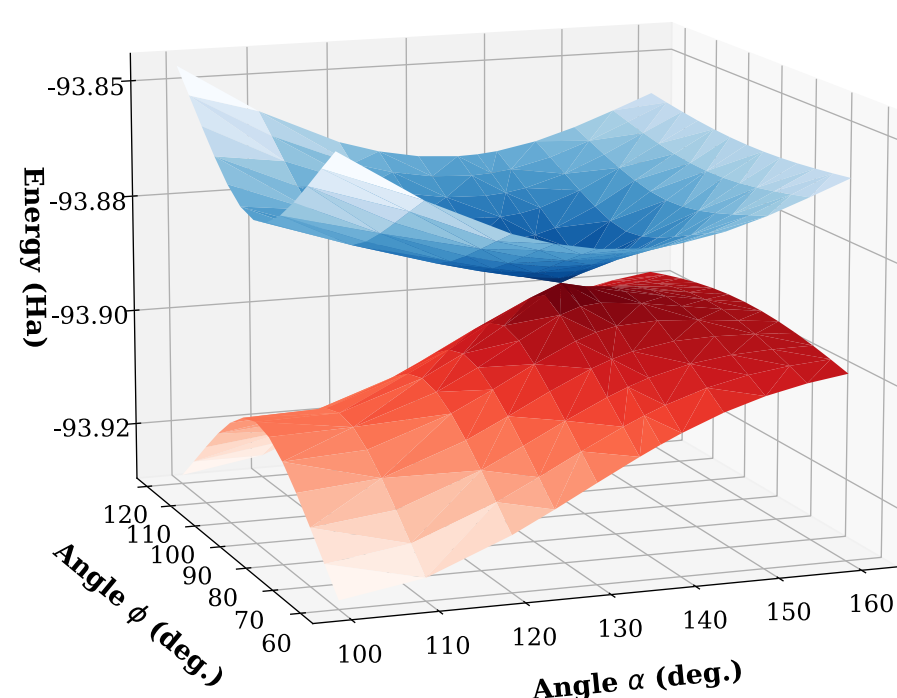
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# Take Home Messages

SA-OO-VQE: Quantum algorithm for photo-chemistry

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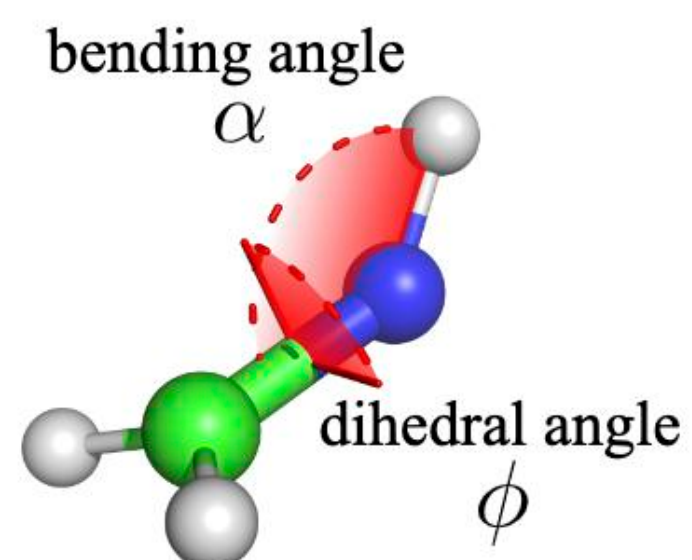


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MECI optimization

**LCQIS**

Quantum Software/Hardware

*Thank you for your attention !*

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